Engineering Mechanics

- **Rigid Body Mechanics**
  - **Statics**
  - **Dynamics**
- **Deformable Body Mechanics**
  - **Strength of Materials**
- **Fluid Mechanics**

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Concept of Stress

- The main objective of the study of mechanics of materials is to provide the future engineer with the means of analyzing and designing various machines and load bearing structures.
- Both the analysis and design of a given structure involve the determination of *stresses* and *deformations*. This chapter is devoted to the concept of stress.

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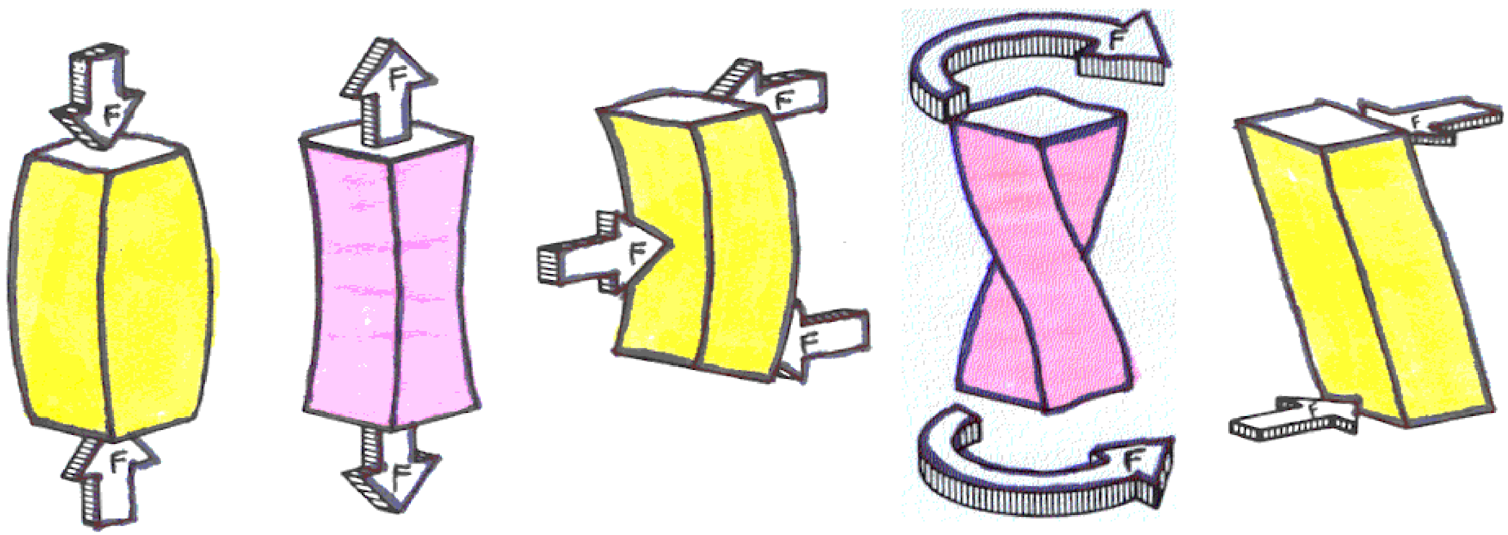
Introduction to Mechanics of Materials

Definition: *Mechanics of materials* is a branch of applied mechanics that deals with the behaviour of solid bodies subjected to various types of loading



Compression ***Tension (stretched)*** ***Bending*** ***Torsion (twisted)*** ***Shearing***

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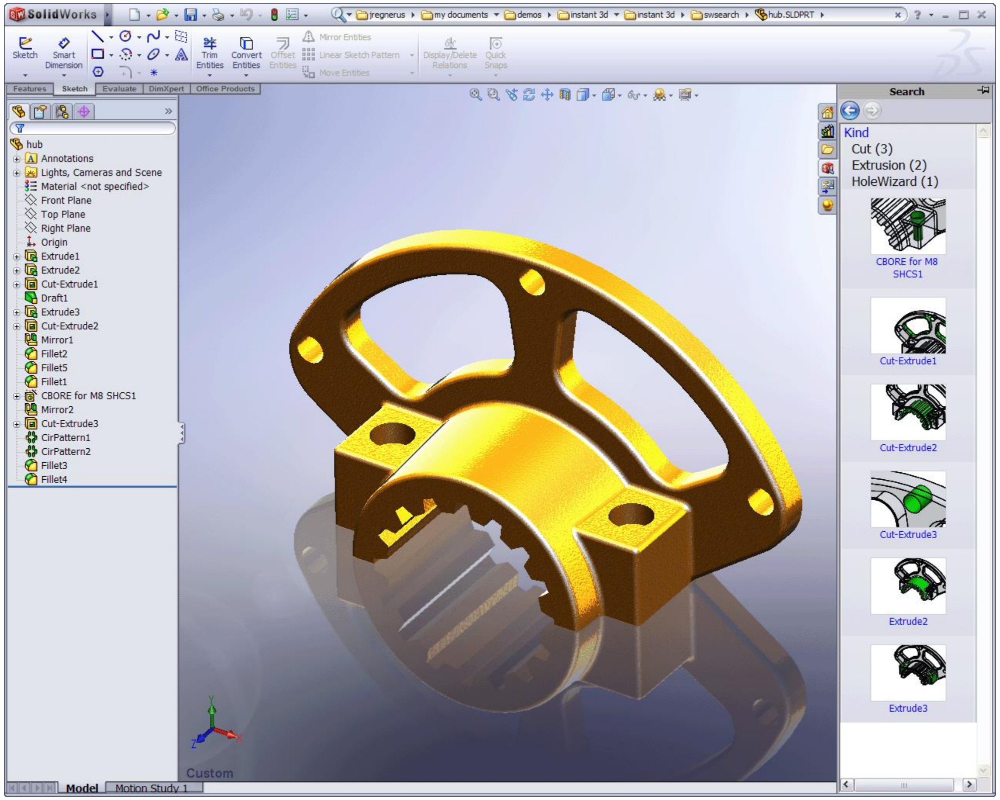
Introduction to Mechanics of Materials

Fundamental concepts

- stress and strain
- deformation and displacement
- elasticity and inelasticity
- load-carrying capacity



Design and analysis of mechanical and structural systems

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Introduction to Mechanics of Materials

- Examination of stresses and strains inside real bodies of finite dimensions that deform under loads

- In order to determine stresses and strains we use:

1. Physical properties of materials

2. Theoretical laws and concepts

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Mechanics of Materials

- External Loads produce Internal Loads
- Internal Loads cause a body to deform
- Internal Loads cause stress
- How much does body deform?
- How much stress?
- Is it Safe at this stress?
- How big should it be so stress is low enough?

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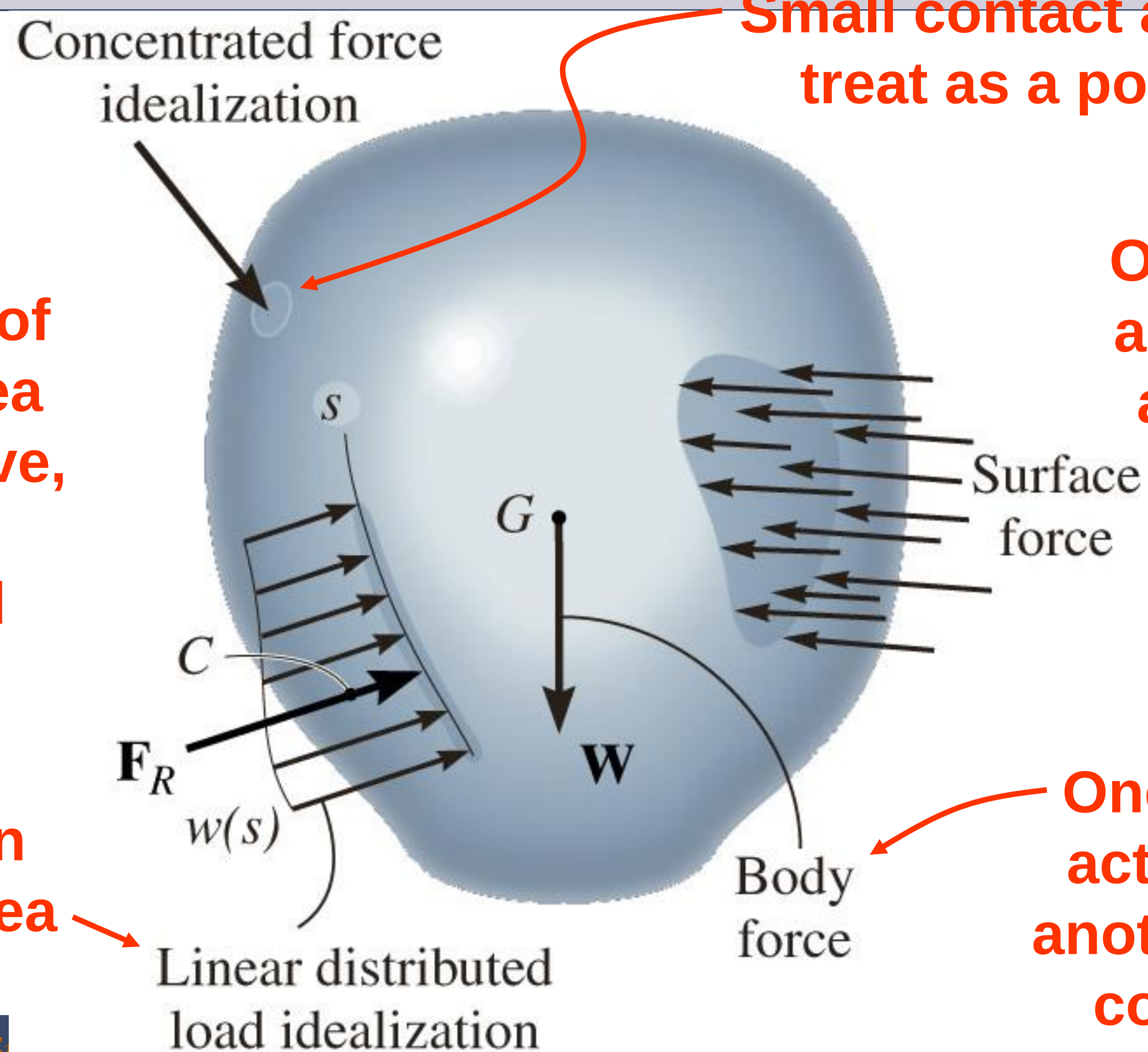
Statics Review: External Loads



Small contact area; treat as a point

F_R is resultant of w(s) = area under curve, acts at centroid

One body acting on another

Acting on narrow area

One body acting on another w/o contact

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External Loads:

- External loads can be *Reaction Loads* or *Applied Loads*!
- Must solve for all unknown external loads (reaction loads) so that internal loads can be solved for!
- Internal loads produce stress, strain, deformation – SofM concepts!

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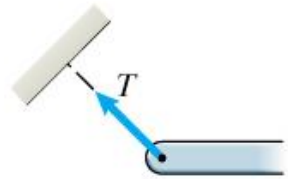
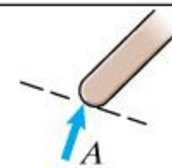
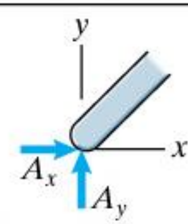
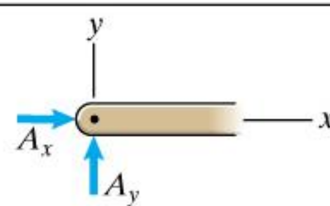
Support Types and Reactions (2D):

Supports	Reactions
 Rope or Cable  Spring	 A Collinear Force
 Contact with a Smooth Surface	 A Force Normal to the Supporting Surface
 Contact with a Rough Surface	 Two Force Components
 Pin Support	 Two Force Components

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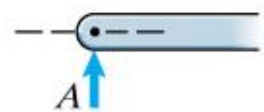
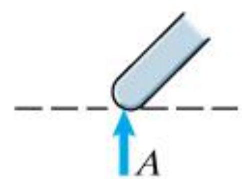
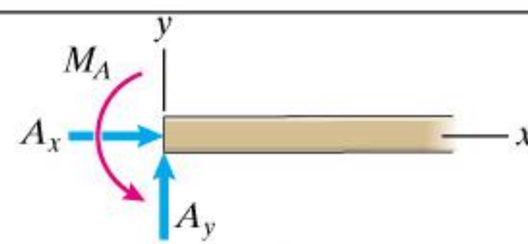
Support Types and Reactions (2D):

Table 5.1

Supports	Reactions
 Roller Support  Equivalents	 A Force Normal to the Supporting Surface
 Constrained Pin or Slider	 A Normal Force
 Fixed (Built-in) Support	 Two Force Components and a Couple

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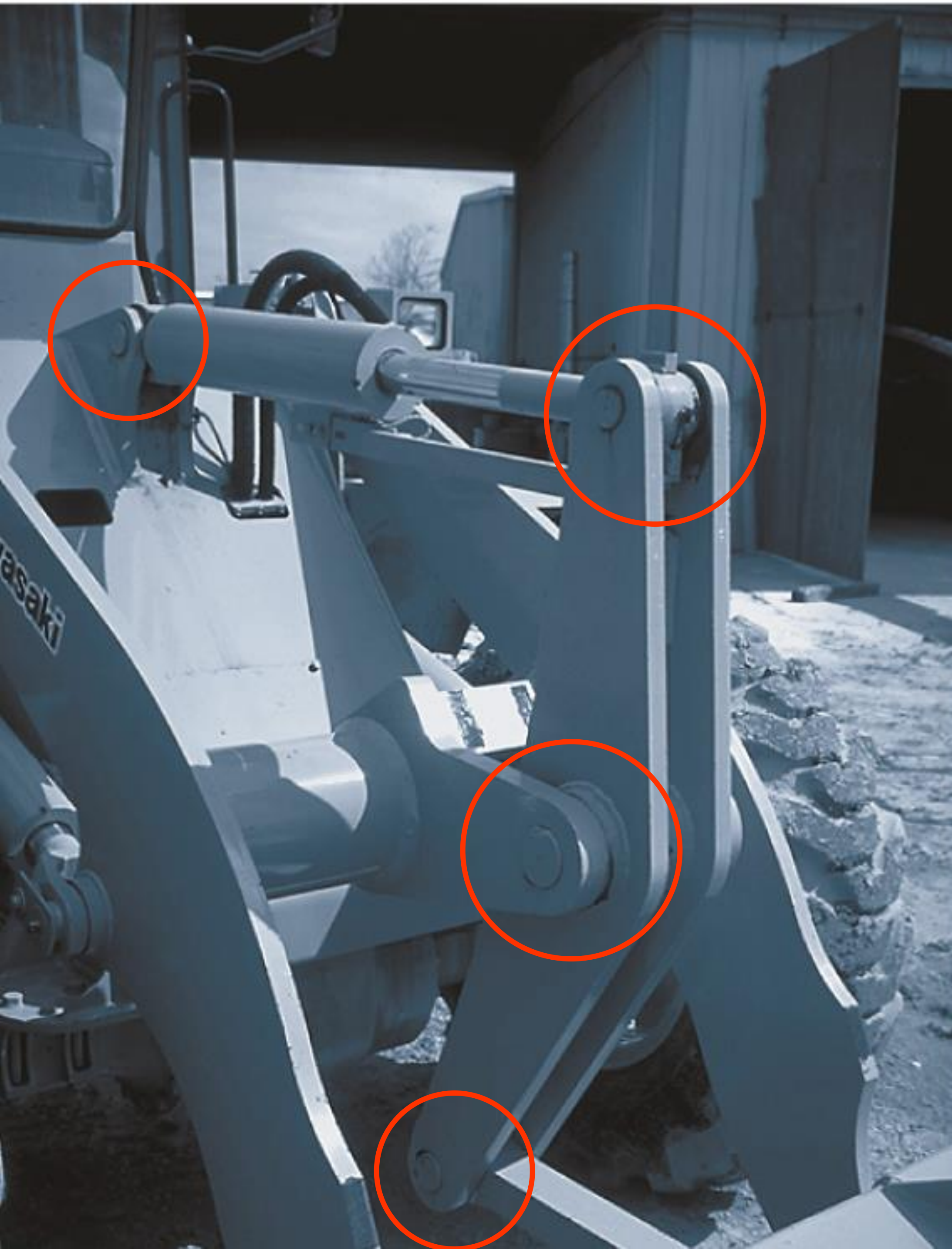

Pin connections allow rotation. Reactions at pins are forces and NOT MOMENTS.

Degrees of Freedom

Static Equilibrium

- Vectors: $\Sigma \mathbf{F} = 0$ $\Sigma \mathbf{M} = 0$

- Coplanar (2D) force systems:

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M_o = 0$$

← **Perpendicular to the plane containing the forces**

- Draw a FBD to account for ALL loads acting on the body.

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STATICS: You need to be able to…

- Draw free-body diagrams,
- Know support types and their corresponding reactions,
- Write and solve equilibrium equations so that unknown forces can be solved for,
- Solve for appropriate internal loads by taking cuts of inspection,
- Determine the centroid of an area,
- Determine the moment of inertia about an axis through the centroid of an area.

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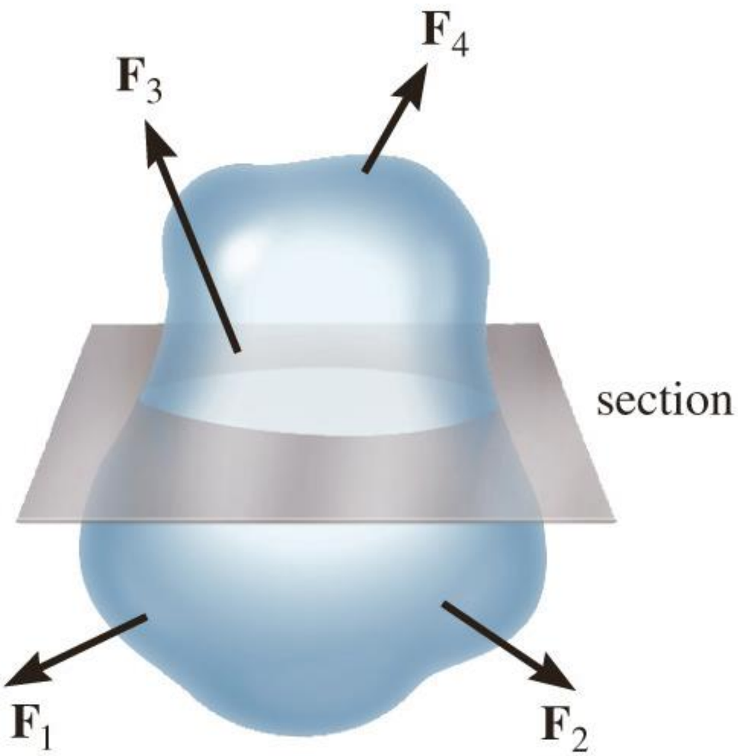
Internal Reactions



- Internal reactions are necessary to hold body together under loading.
- Method of sections - make a cut through body to find internal reactions at the point of the cut.

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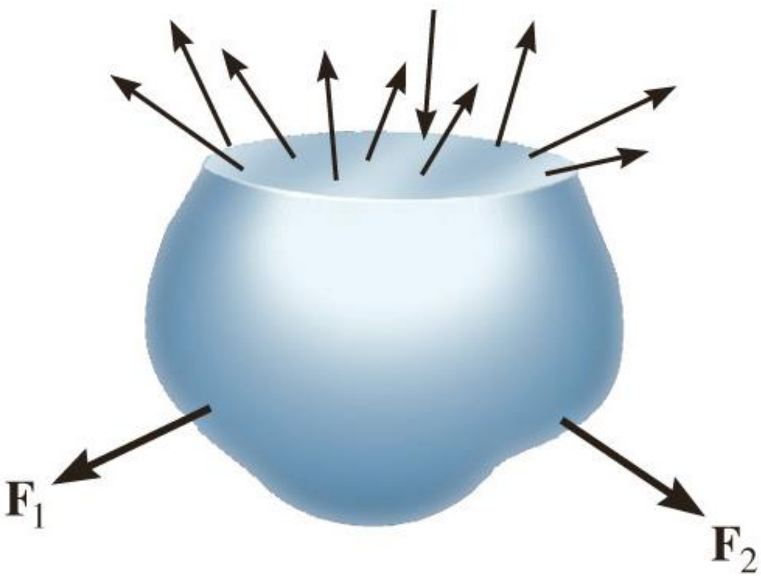
FBD After Cut



- Separate the two parts and draw a FBD of either side
- Use equations of equilibrium to relate the external loading to the internal reactions.

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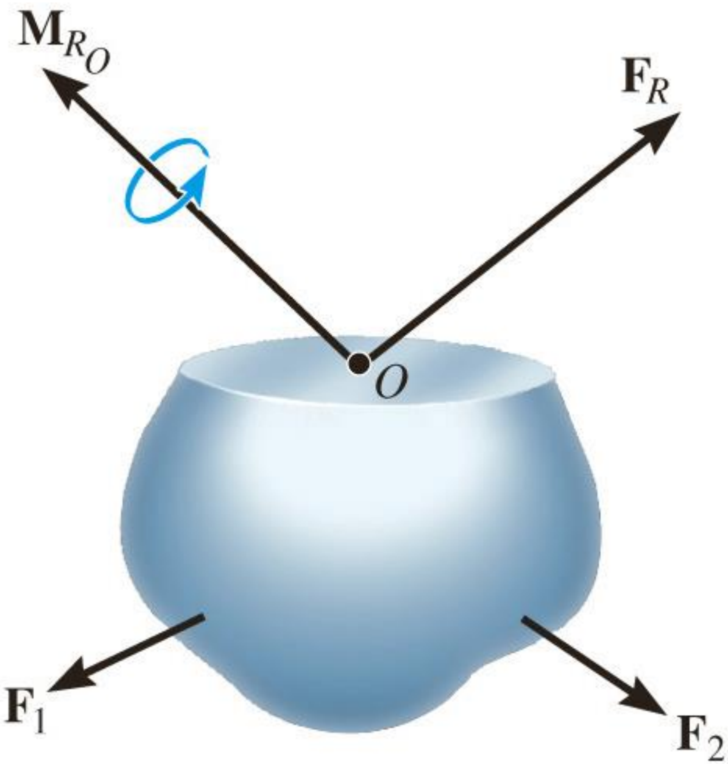
Resultant Force and Moment



- Point O is taken at the centroid of the section.
- If the member (body) is long and slender, like a rod or beam, the section is generally taken perpendicular to the longitudinal axis.
- Section is called the cross section.

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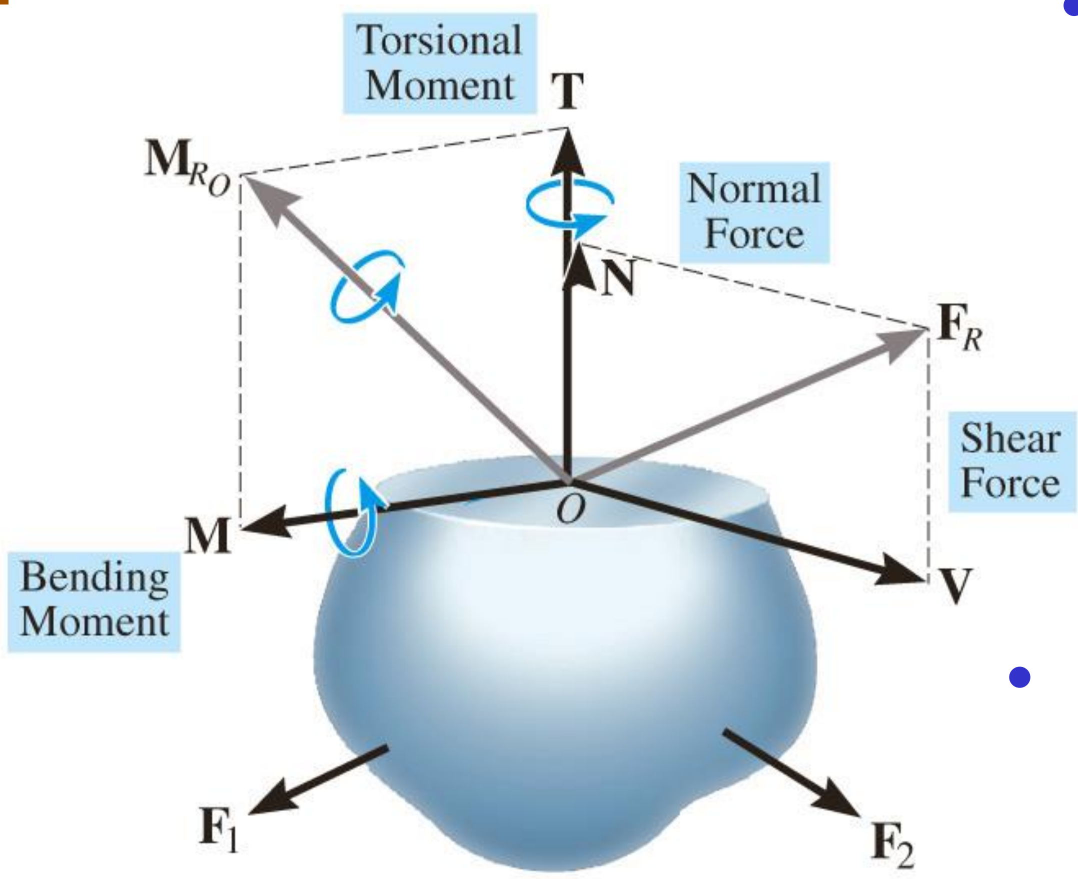
Components of Resultant



- Components are found perpendicular & parallel to the section plane.
- Internal reactions are used to determine stresses.

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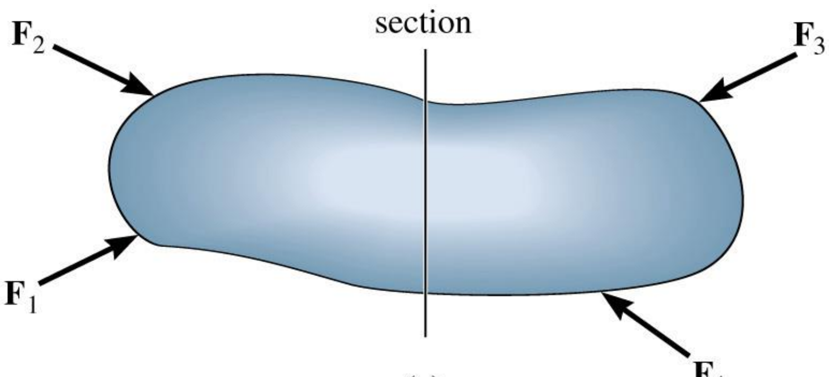
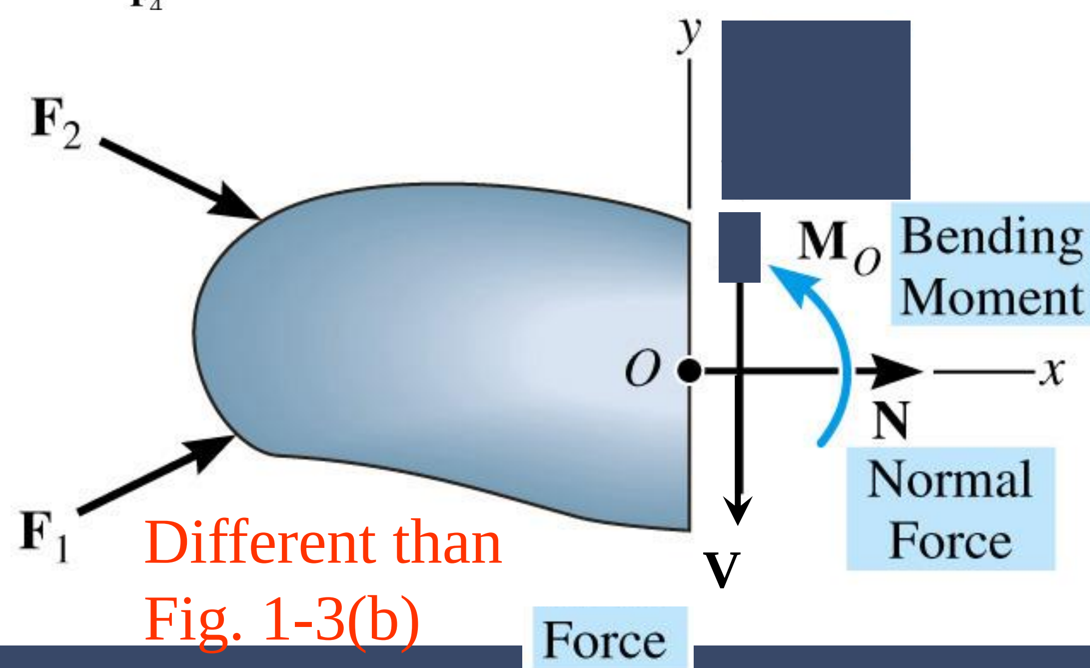
Coplanar Force System



(a)

Start with internal system of forces as shown below to get proper signs for V, N and M.



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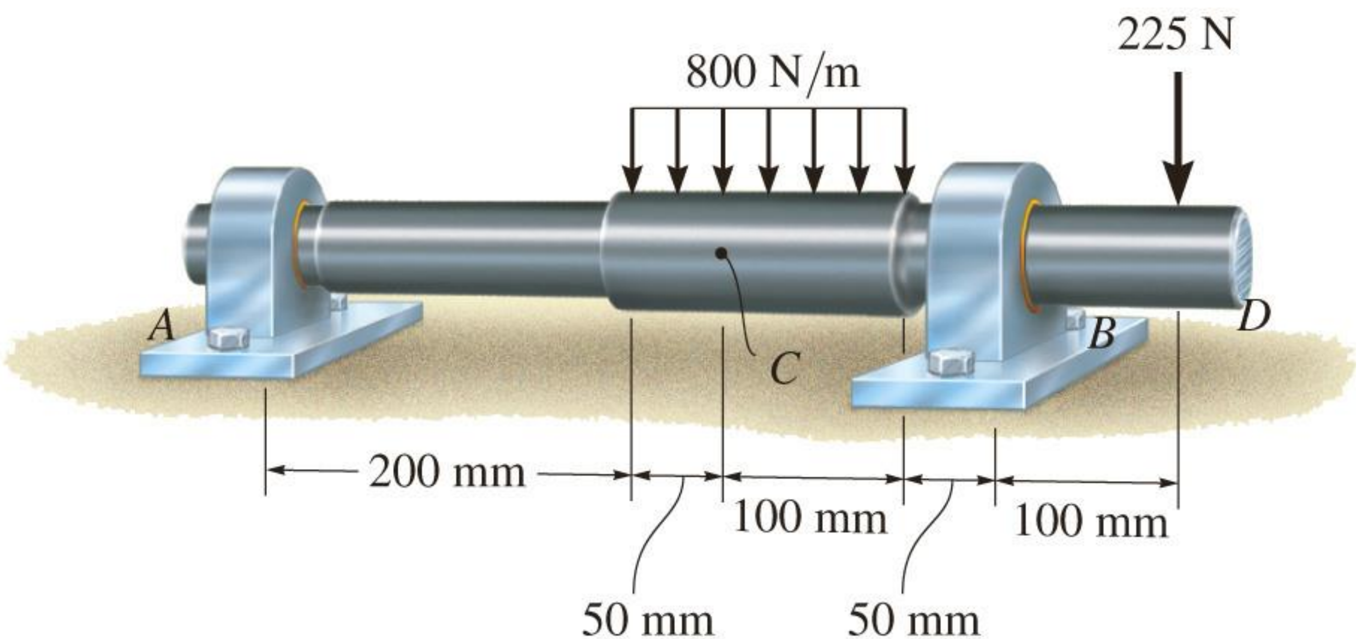
Example: Find the vertical reactions at A and B for the shaft shown.



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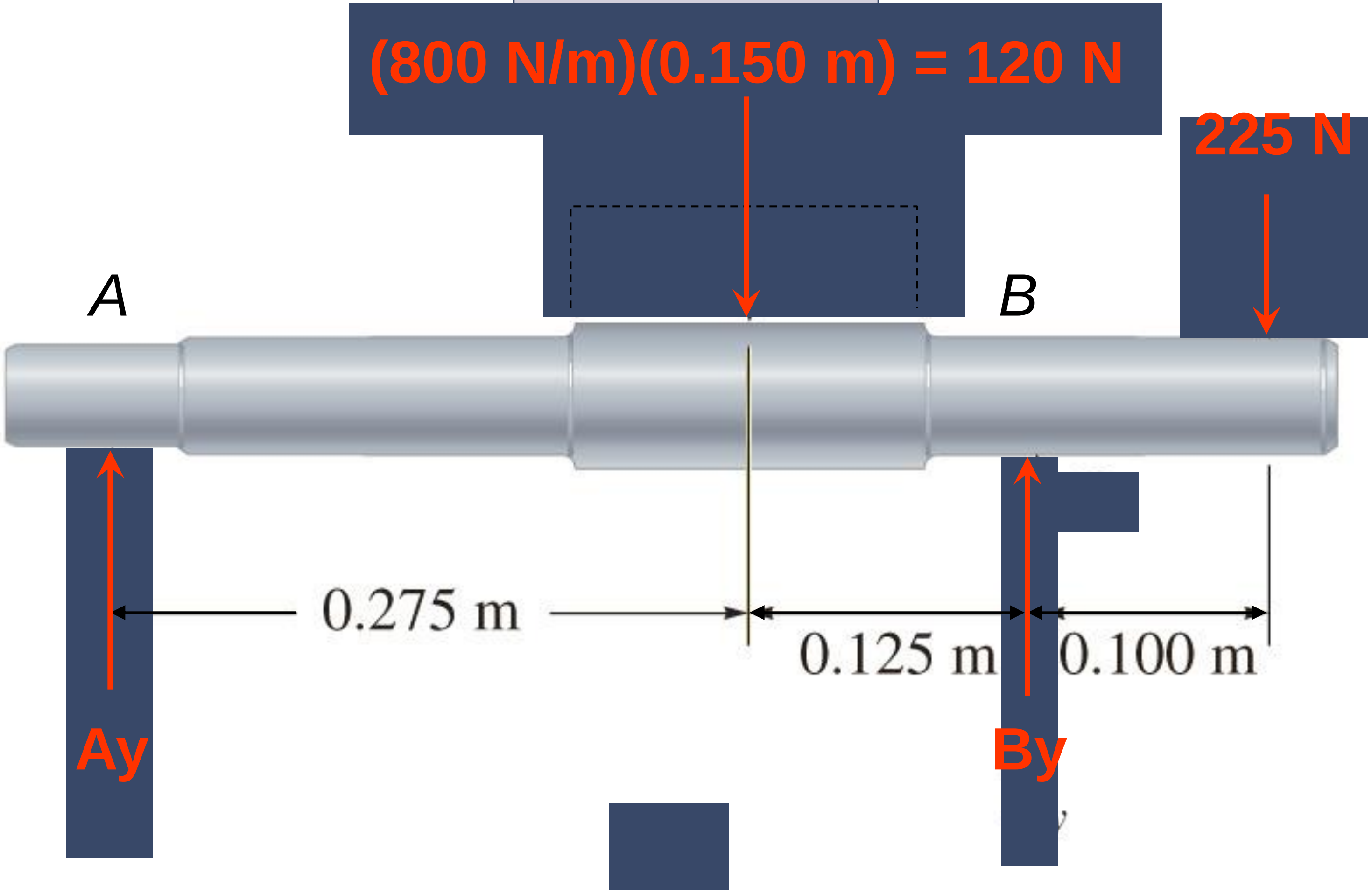
FBD



Comment on dashed line around the distributed load.

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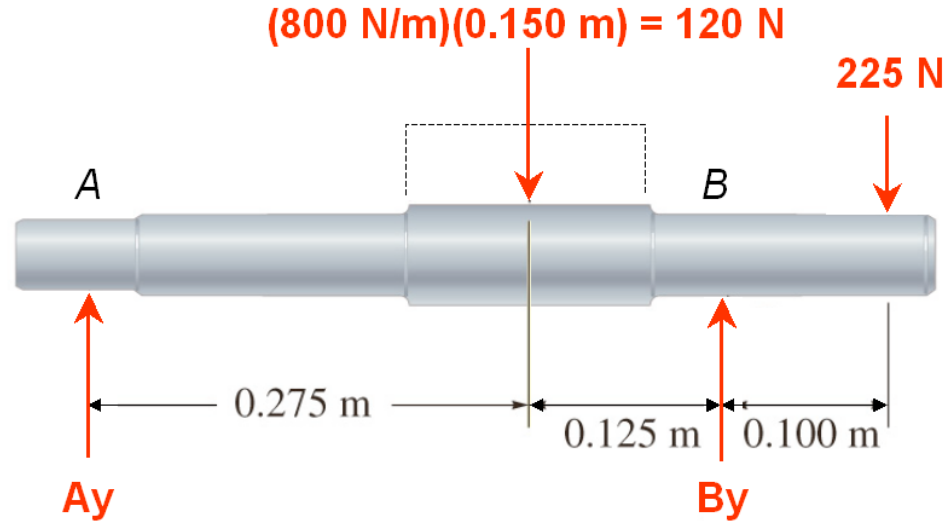
Equilibrium Equations



$$\circlearrowright + \Sigma M_A = 0 = .400\text{m}(B_y) - 120\text{N}(.275\text{m}) - 225\text{N}(.500\text{m})$$

$$B_y = \frac{-120\text{N}(.275\text{m}) - 225\text{N}(.500\text{m})}{-.400\text{m}}$$

$$B_y = \mathbf{363.75N} \uparrow$$

$$+\uparrow \Sigma Fy = 0 = Ay - 120\text{N} + 363.75\text{N} - 225\text{N}$$

$$A_y = -18.75\text{N}$$

$$A_y = \mathbf{18.75N} \downarrow$$

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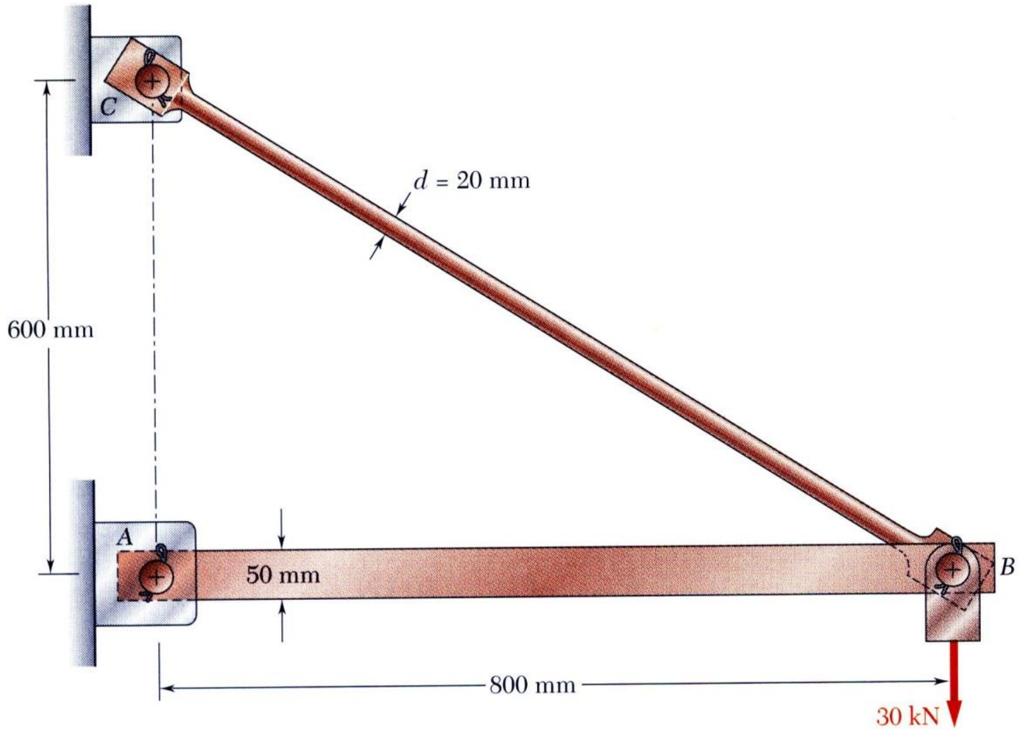
Review of Statics



- The structure is designed to support a 30 kN load
- The structure consists of a boom and rod joined by pins (zero moment connections) at the junctions and supports
- Perform a static analysis to determine the internal force in each structural member and the reaction forces at the supports

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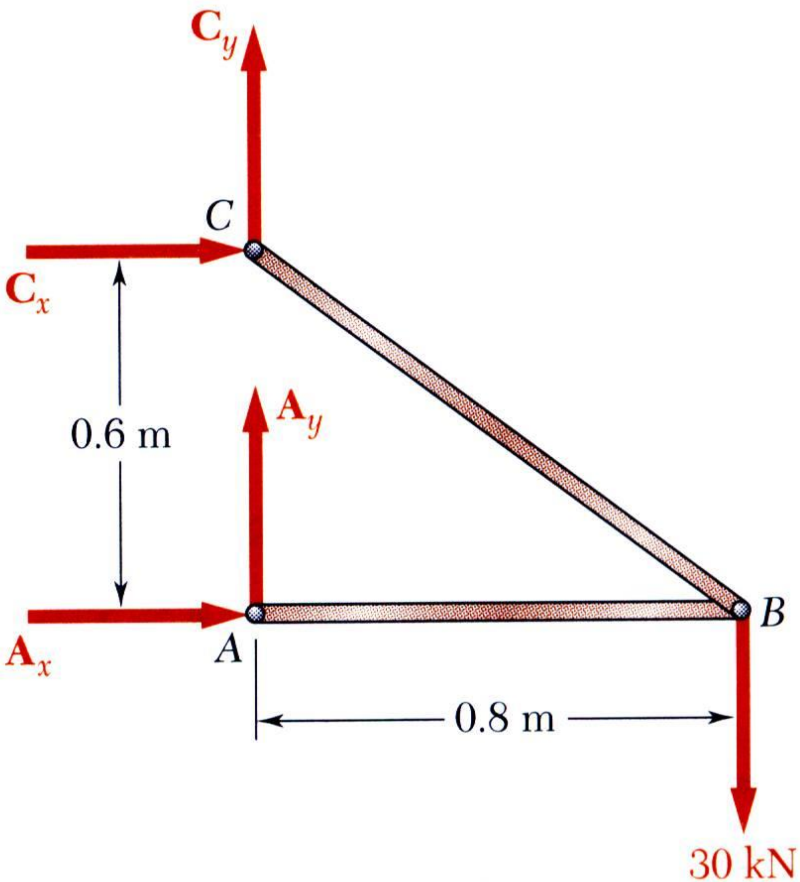
Structure Free-Body Diagram



- Structure is detached from supports and the loads and reaction forces are indicated
- Conditions for static equilibrium:

$$\sum M_C = 0 = A_x(0.6\,\mathrm{m}) - (30\,\mathrm{kN})(0.8\,\mathrm{m})$$

$$A_x = 40\,\mathrm{kN}$$

$$\sum F_x = 0 = A_x + C_x$$

$$C_x = -A_x = -40\,\mathrm{kN}$$

$$\sum F_y = 0 = A_y + C_y - 30\,\mathrm{kN} = 0$$

$$A_y + C_y = 30\,\mathrm{kN}$$

- A_y and C_y can not be determined from these equations

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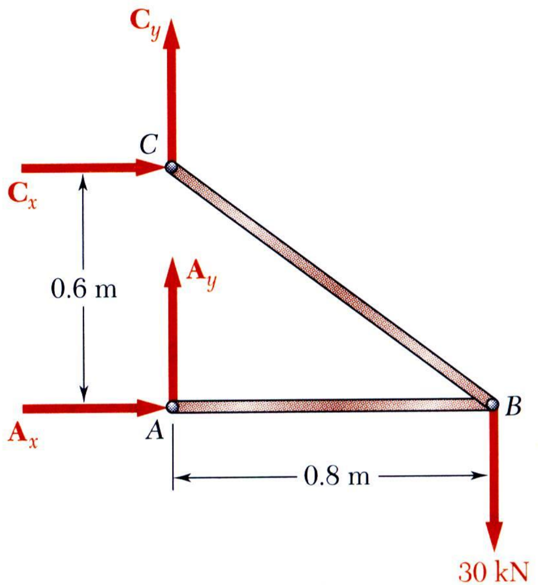
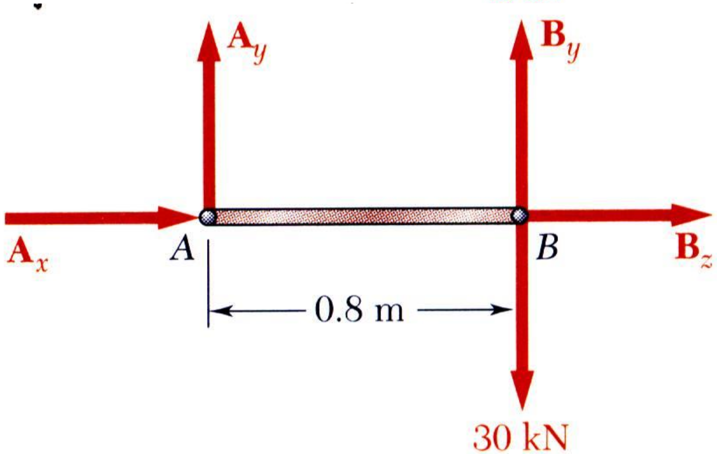
Component Free-Body Diagram





- In addition to the complete structure, each component must satisfy the conditions for static equilibrium

- Consider a free-body diagram for the boom:

  $$\sum M_B = 0 = -A_y(0.8\,\text{m})$$

  $$A_y = 0$$

  substitute into the structure equilibrium equation

  $$C_y = 30\,\text{kN}$$

- Results:

  $$A = 40\,\text{kN} \rightarrow \quad C_x = 40\,\text{kN} \leftarrow \quad C_y = 30\,\text{kN} \uparrow$$

  Reaction forces are directed along boom and rod

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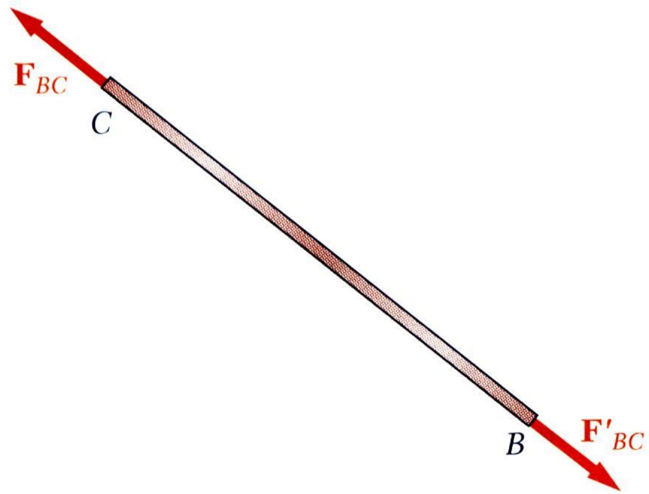
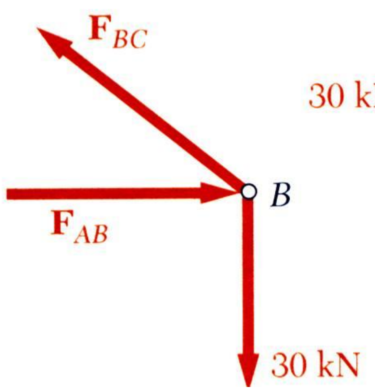
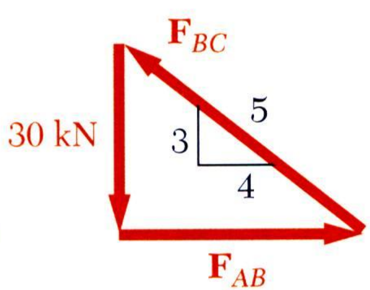
Method of Joints







- The boom and rod are 2-force members, i.e., the members are subjected to only two forces which are applied at member ends
- For equilibrium, the forces must be parallel to to an axis between the force application points, equal in magnitude, and in opposite directions
- Joints must satisfy the conditions for static equilibrium which may be expressed in the form of a force triangle:

$$\sum \vec{F}_B = 0$$

$$\frac{F_{AB}}{4} = \frac{F_{BC}}{5} = \frac{30\,\text{kN}}{3}$$

$$F_{AB} = 40\,\text{kN} \qquad F_{BC} = 50\,\text{kN}$$

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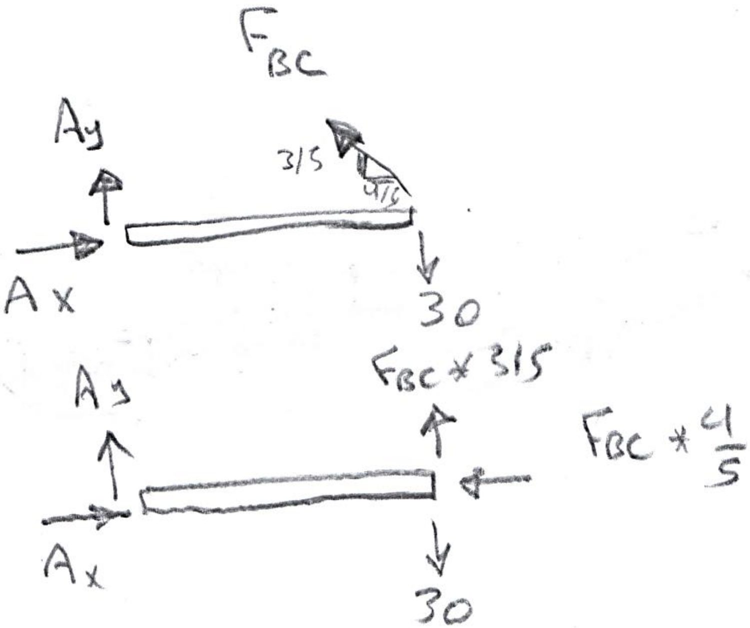
Quick answer is welcomed!



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$

$A_y + F_{BC} * \frac{3}{5} - 30 = 0$

$A_x = F_{BC} * 4/5$

$F_{BC} * \frac{3}{5} = 30$

$F_{BC} = 50\ \text{KN}$

$A_x = 40\ \text{KN}$

$A_y = 0$

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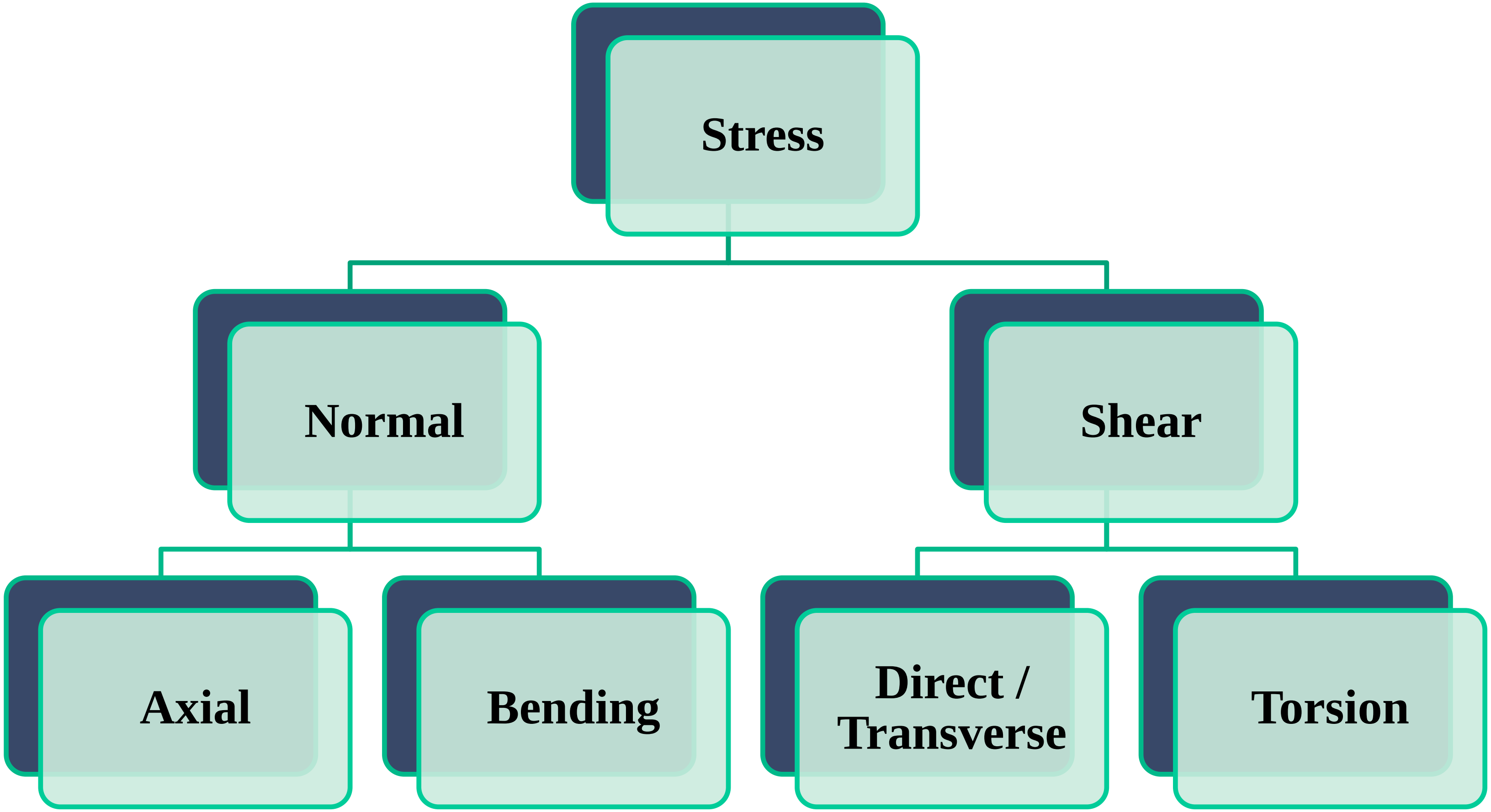
Stresses types



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Normal stresses

$$\sigma_{Normal} = \sigma_{Axial} + \sigma_{Bending}$$

$$\sigma_{Axial} = \frac{P}{A} = \frac{\text{Normal load}}{\text{Cross sectional area}}$$

$$\sigma_{Bending} = \frac{M.c}{I}$$

Where

M: Bending moment at a specific point (if not mentioned, take the maximum value).

C: distance from centroid to the point you calculate stress at it, is calculated from the cross section (if not mentioned, take the maximum value).

I: Moment of inertia of the cross section.

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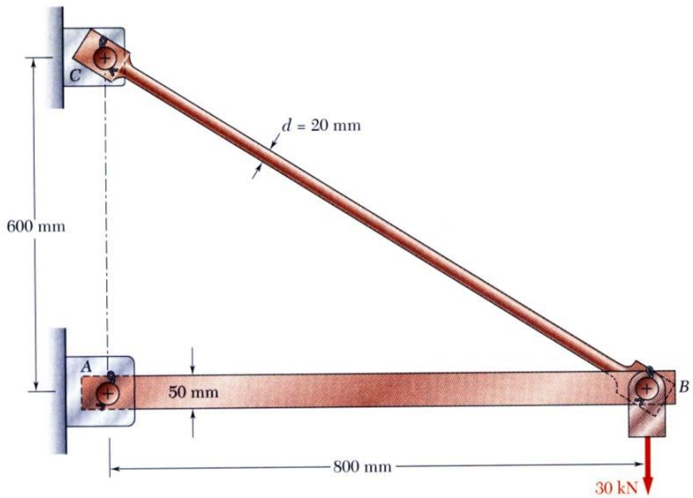
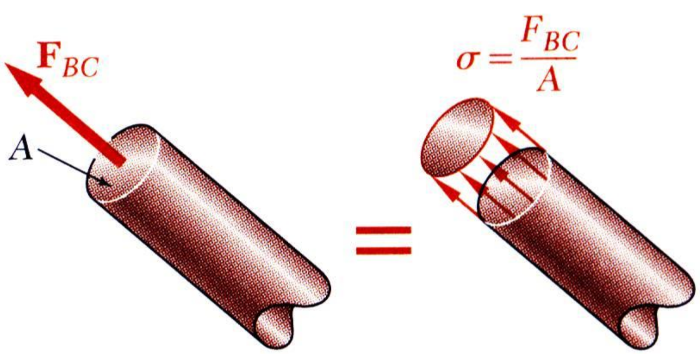
Stress Analysis



$d_{BC} = 20$ mm



Can the structure safely support the 30 kN load?

- From a statics analysis

  $F_{AB} = 40$ kN (compression)

  $F_{BC} = 50$ kN (tension)

- At any section through member BC, the internal force is 50 kN with a force intensity or stress of

$$\sigma_{BC} = \frac{P}{A} = \frac{50 \times 10^3 \text{ N}}{314 \times 10^{-6} \text{ m}^2} = 159 \text{ MPa}$$

- From the material properties for steel, the allowable stress is

$$\sigma_{\text{all}} = 165 \text{ MPa}$$

- Conclusion: the strength of member *BC* is adequate

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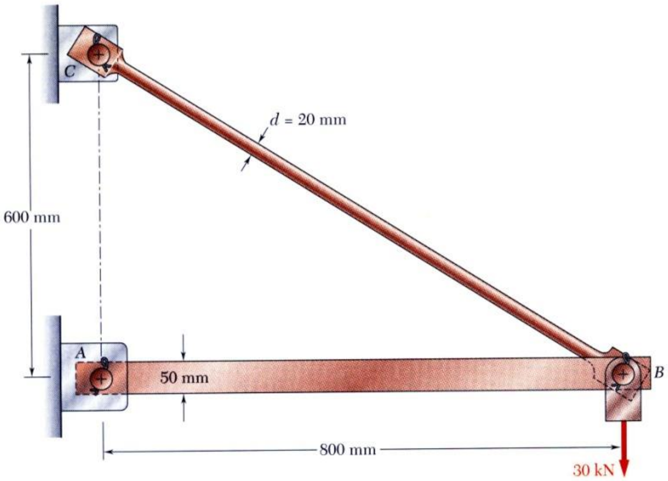
Design



- Design of new structures requires selection of appropriate materials and component dimensions to meet performance requirements
- For reasons based on cost, weight, availability, etc., the choice is made to construct the rod from aluminum (σ_{all}= 100 MPa). What is an appropriate choice for the rod diameter?

$$\sigma_{all} = \frac{P}{A} \qquad A = \frac{P}{\sigma_{all}} = \frac{50\times10^3\,\mathrm{N}}{100\times10^6\,\mathrm{Pa}} = 500\times10^{-6}\,\mathrm{m}^2$$

$$A = \pi\frac{d^2}{4}$$

$$d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4\left(500\times10^{-6}\,\mathrm{m}^2\right)}{\pi}} = 2.52\times10^{-2}\,\mathrm{m} = 25.2\,\mathrm{mm}$$

- An aluminum rod 26 mm or more in diameter is adequate

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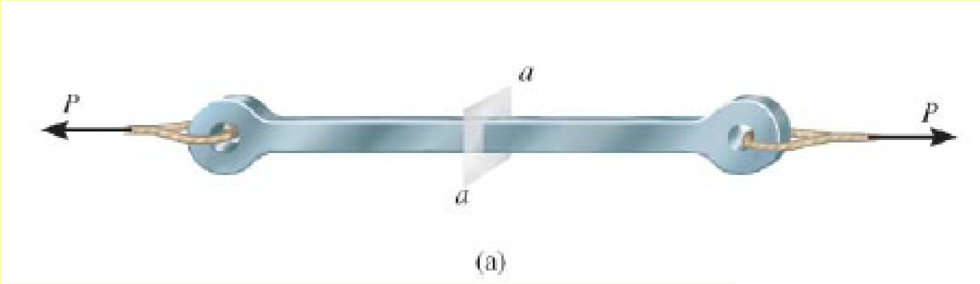
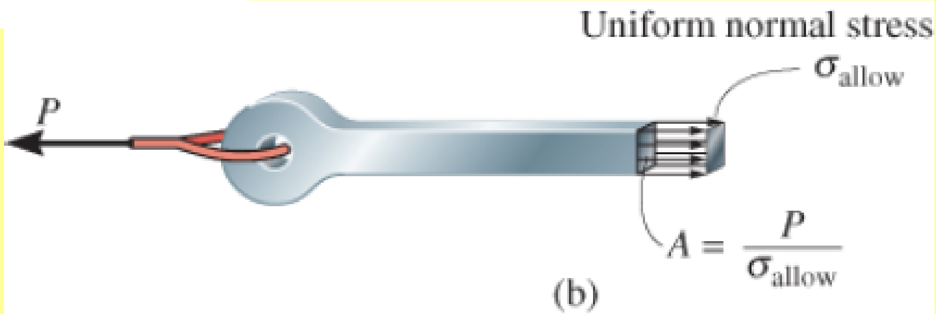
Tension stress

Cross-sectional area of a tension member



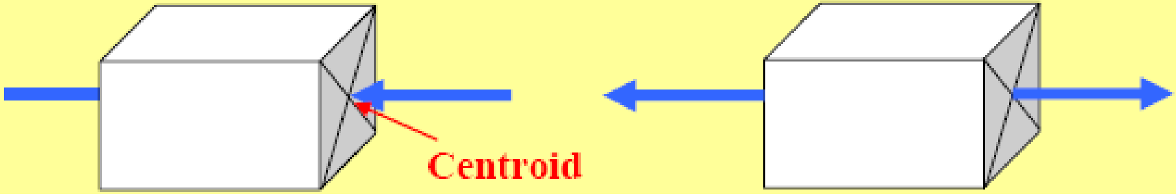
(a)



(b)

Condition:

The force has a **line of action** that passes through the **centroid of the cross section**.



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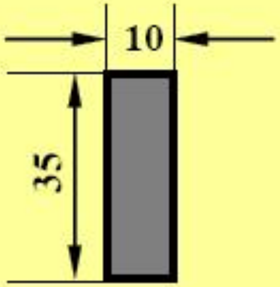
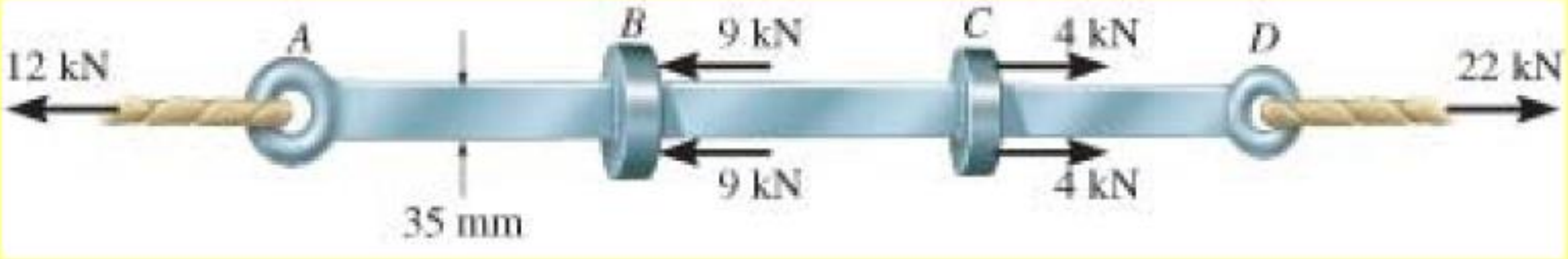
Tension stress

Determine the max. average normal stress in the bar when subjected to the loadings as shown below.

Bar width = 35 mm, thickness = 10 mm





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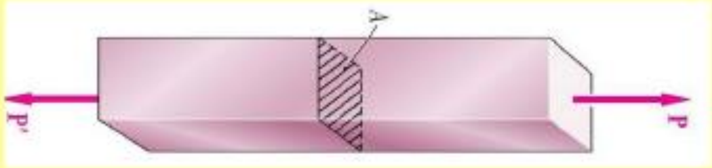
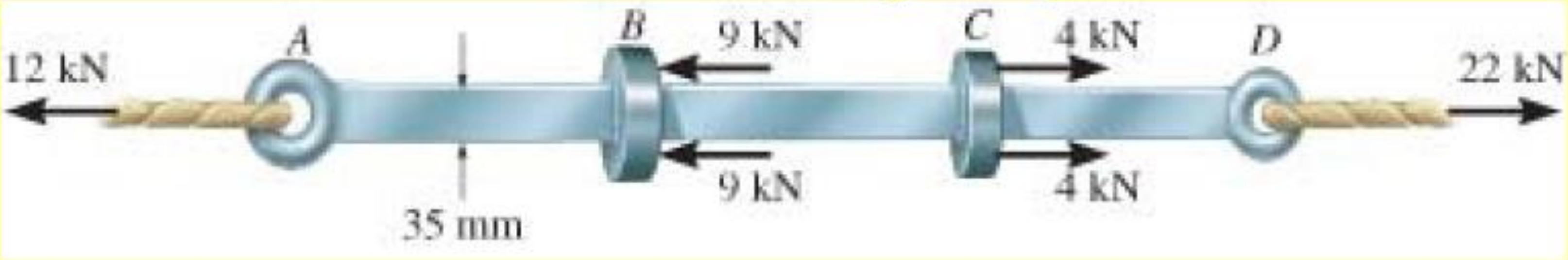
Tension stress

Maximum average normal stress

- IF the internal force **P and x-sec area A** were *constant* along the longitudinal axis of the bar, then **normal stress σ = P/A** is also *constant*



- If the bar is subjected to several external loads along its axis, change in x-sectional area may occur. Thus, it is important to find the maximum average normal stress,



- To determine that, we need to find the *location* where ratio *P*/*A* is a maximum

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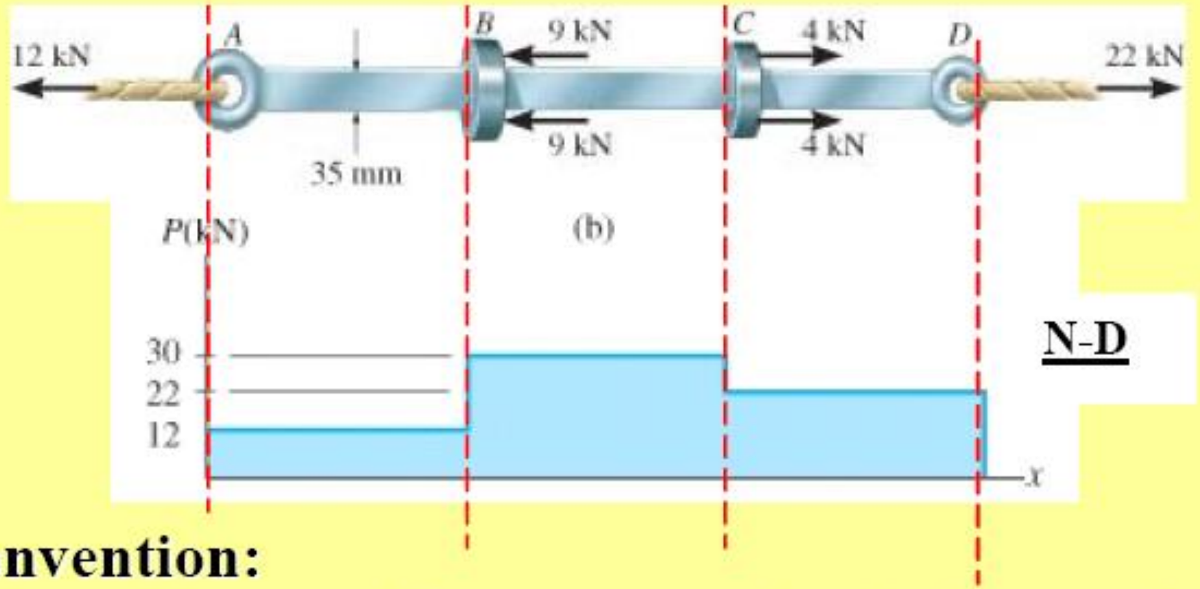
Tension stress

Maximum average normal stress

- Draw an *axial or normal force diagram* **N-D**



Sign convention:

- *P* is positive (+) if it causes tension in the member
- *P* is negative (−) if it causes compression

- **Identify the maximum** average normal stress from the plot

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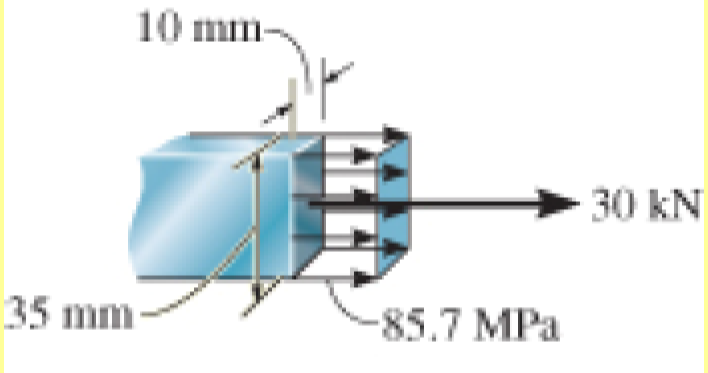
Tension stress

Maximum Average normal stress

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{30(10^3)\text{ N}}{(0.035\text{ m})(0.010\text{ m})} = \mathbf{85.7\ MPa}$$



(d)

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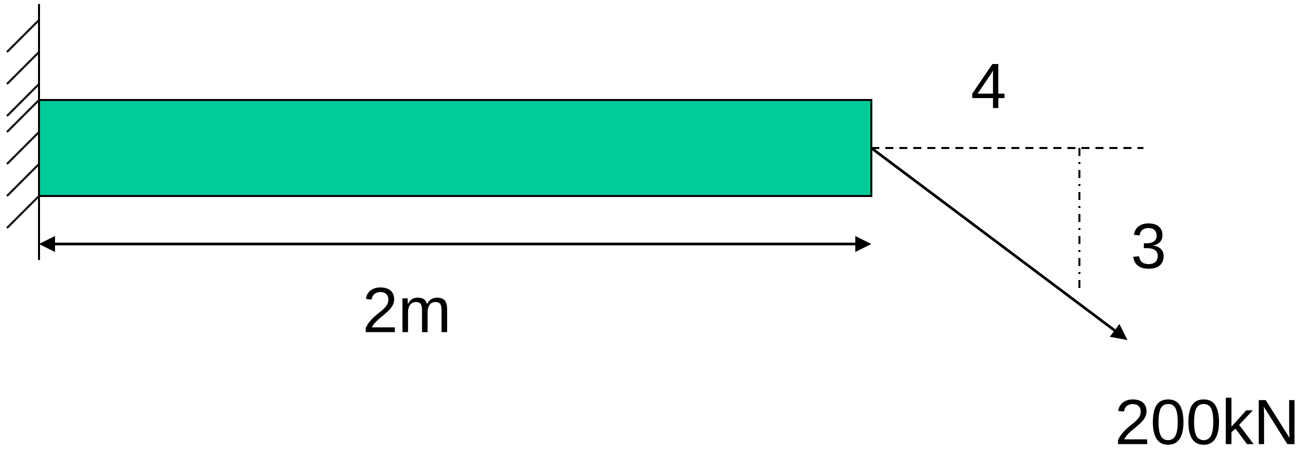
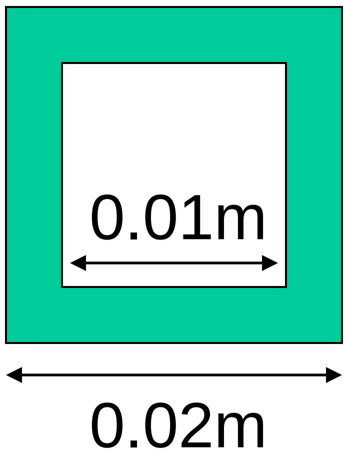
Combined stress example (Normal stresses)



A steel beam with a tensile strength of 700 MPA is loaded as shown. Assuming that the beam is made from hollow square tubing with the dimensions shown will the loading in the x direction exceed the failure stress?



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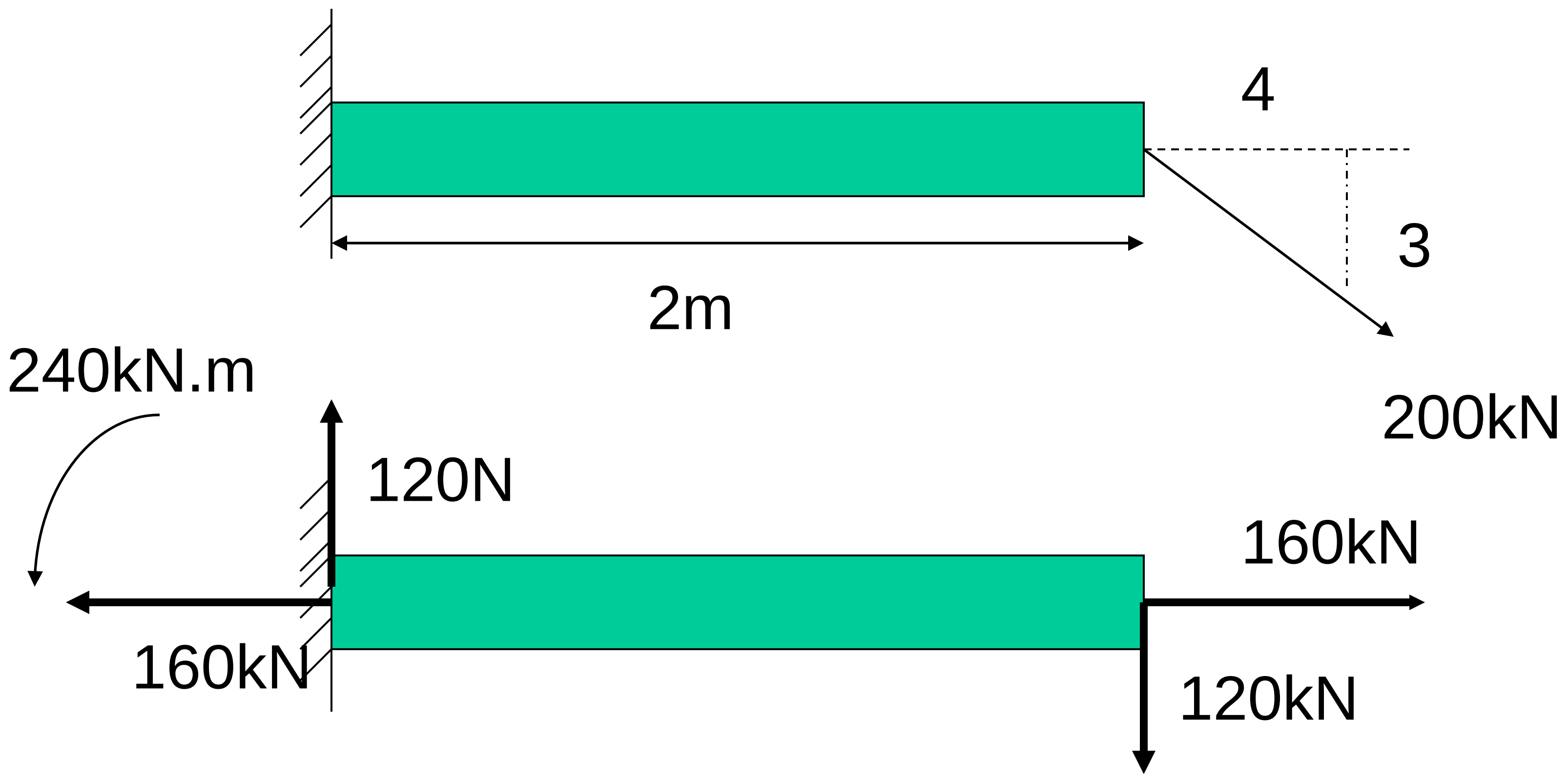
Step 1: Free body diagram



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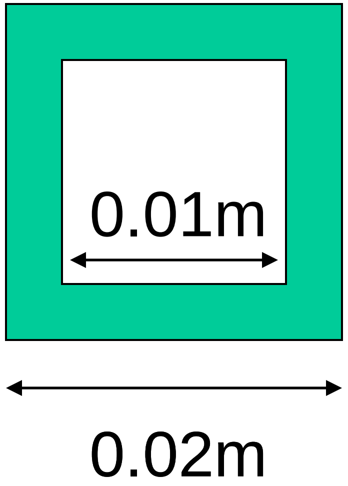
Step 2: Calculate moment of inertia

$I = 1/12 \times (0.02^4) - 1/12 \times (0.01^4)\ m^4$

$= 1.25 \times 10^{-8}\ m^4$

$A = 0.02^2 - 0.01^2\ m^2$

$= 0.0003\ m^2$



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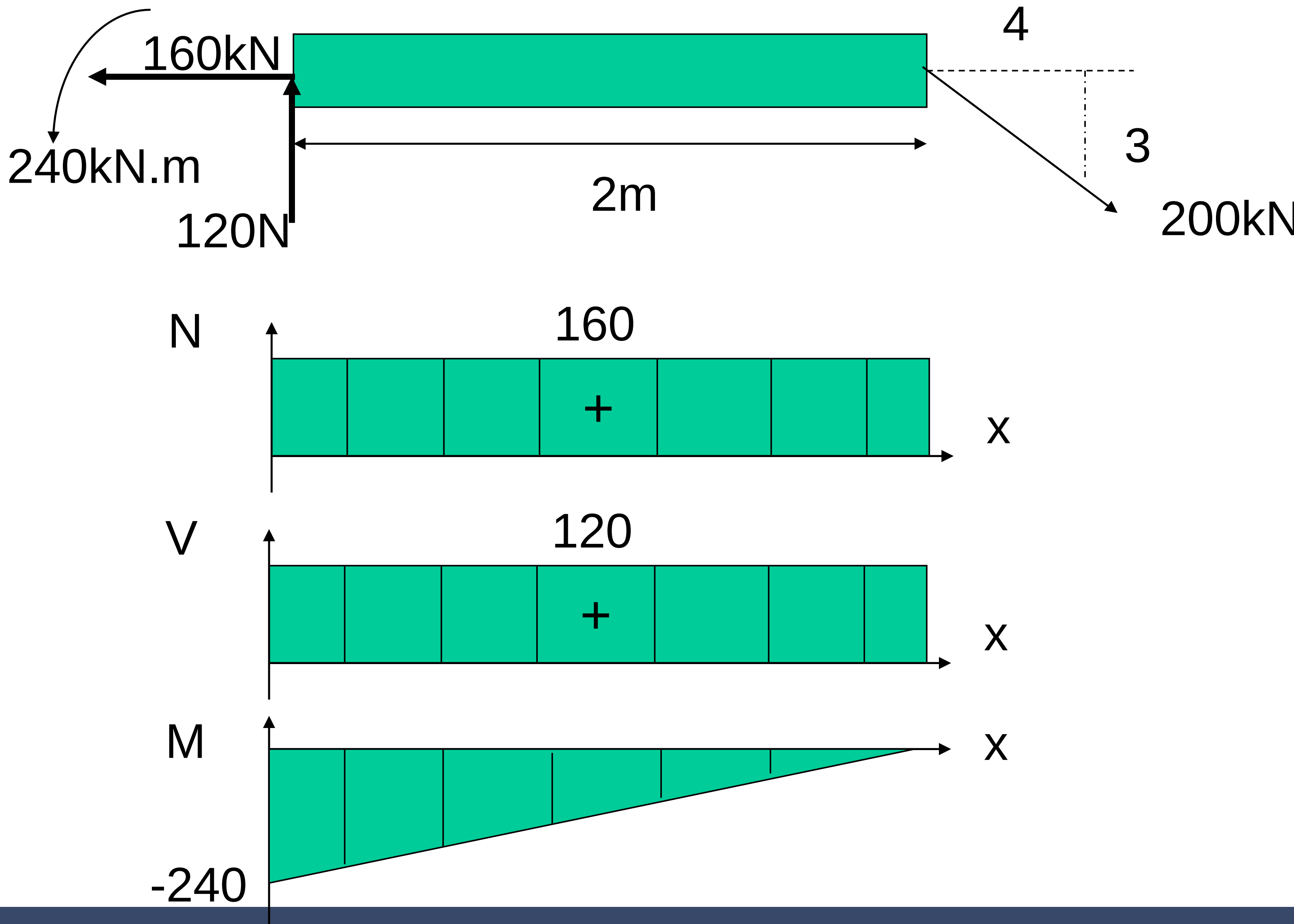
Step 3: Normal, Shear and moment diagrams



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Step 4: Calculation of maximum tensile stress

- Stress due to axial loading

$$\sigma_{axial} = \frac{F}{A} = \frac{160}{0.0003} kPa = 533.33 MPa$$

- Stress due to bending

$$\sigma_{bend} = \frac{Mc}{I} = \frac{240 \times 0.01}{1.25 \times 10^{-8}} kPa = 1920 MPa$$

ANS: Total stress greater than failure stress therefore failure will occur

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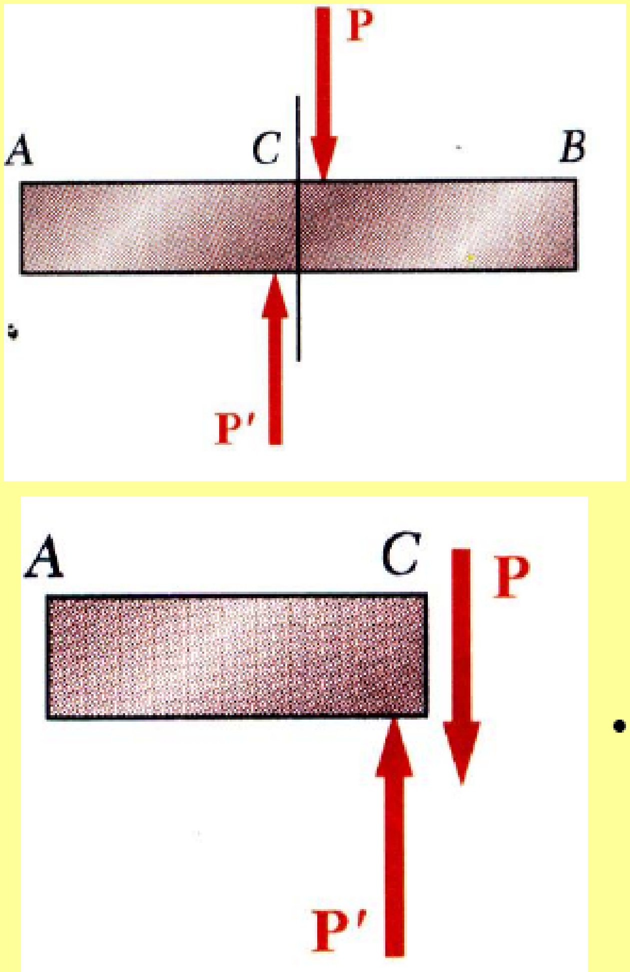
Direct shear stress



Applied transverse forces (P and P`) to the member AB produce internal forces at section *C* which are called *shearing* forces.

- The corresponding average shear stress is,

$$\tau_{\text{ave}} = \frac{P}{A}$$

- Shear stress distribution varies from zero at the member surfaces to maximum values that may be much larger than the average value.

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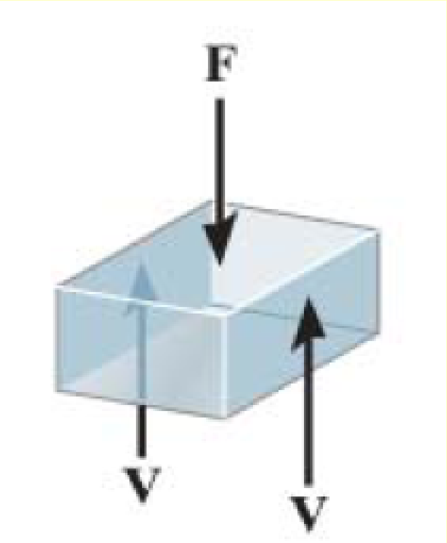
Direct shear stress

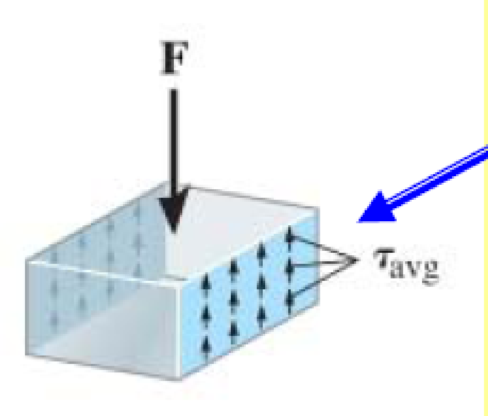
Average shear stress over each section is:



$$\tau_{avg} = \frac{P}{A_{sh}}$$

τ_{avg} = average shear stress at section, assumed to be same at each point on the section

A_{sh} = sheared area of section



This loading case is known as direct or simple shear

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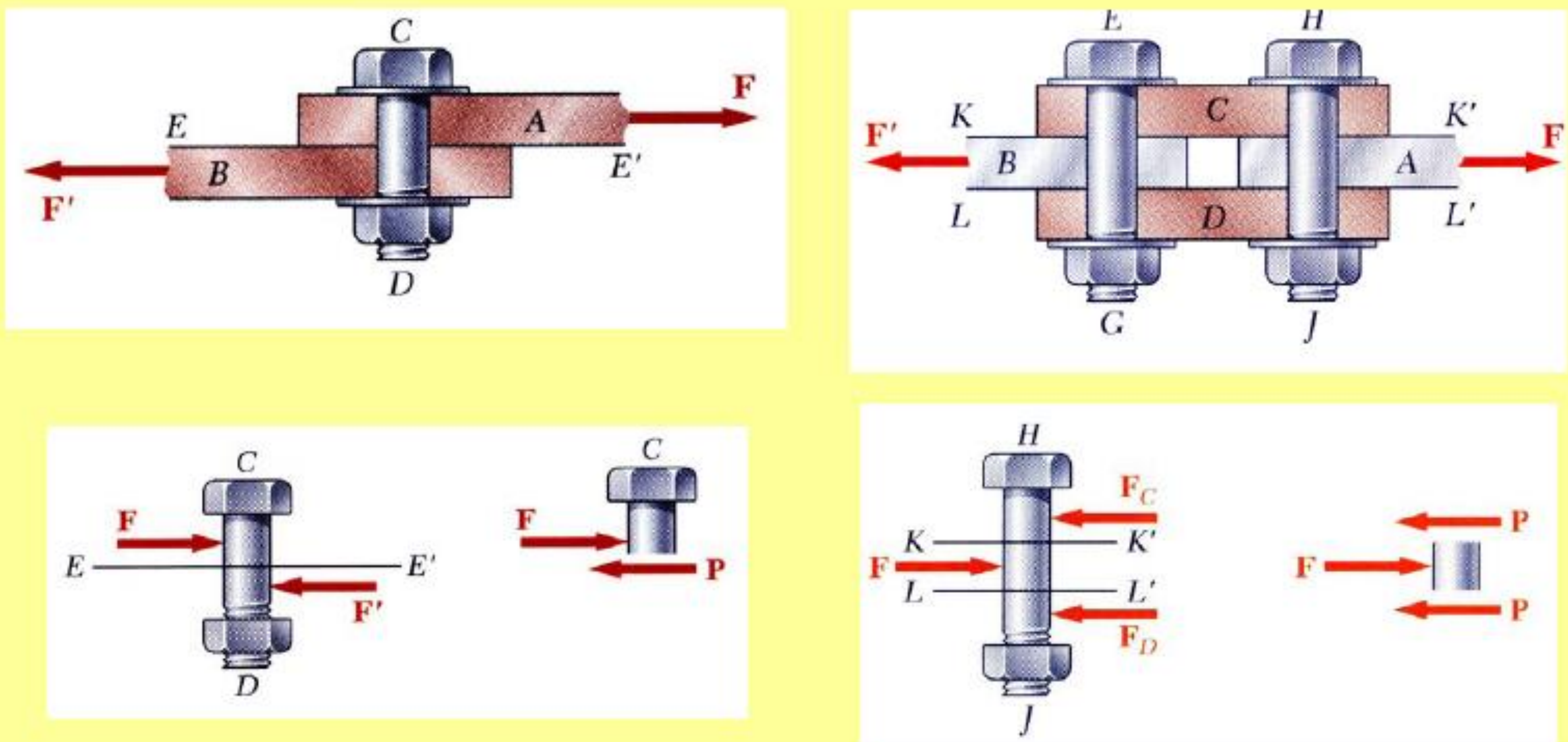
Direct shear stress

Single & Double Shear Stresses

Single Shear | Double Shear



$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{A}$$

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{F}{2A}$$

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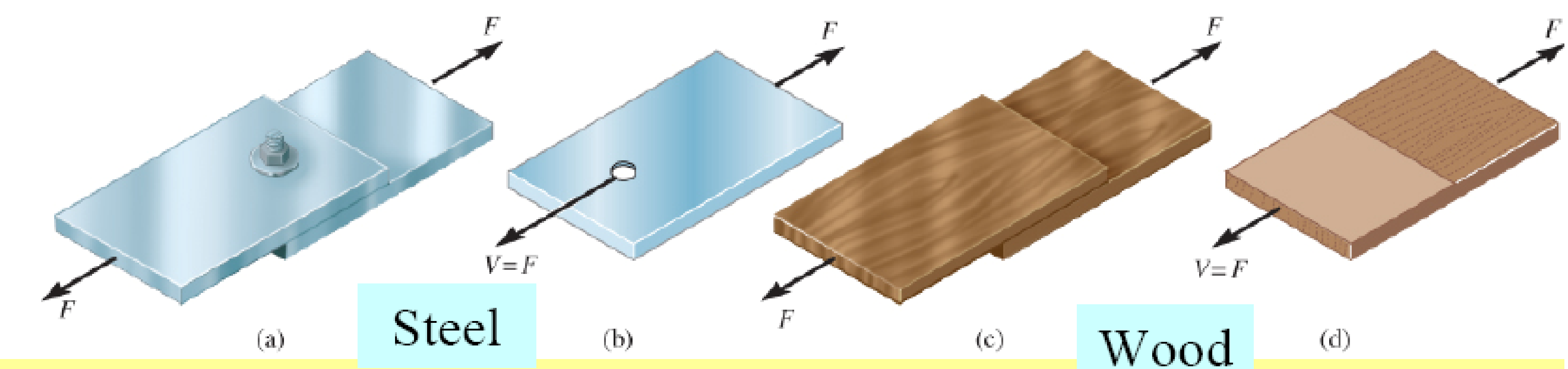
Direct shear stress

Single shear (Single Shear Connection)

- Steel and wood joints shown below are examples of *single-shear connections*, also **known as lap joints.**
- Since we assume members are thin, there are no moments caused by *F*



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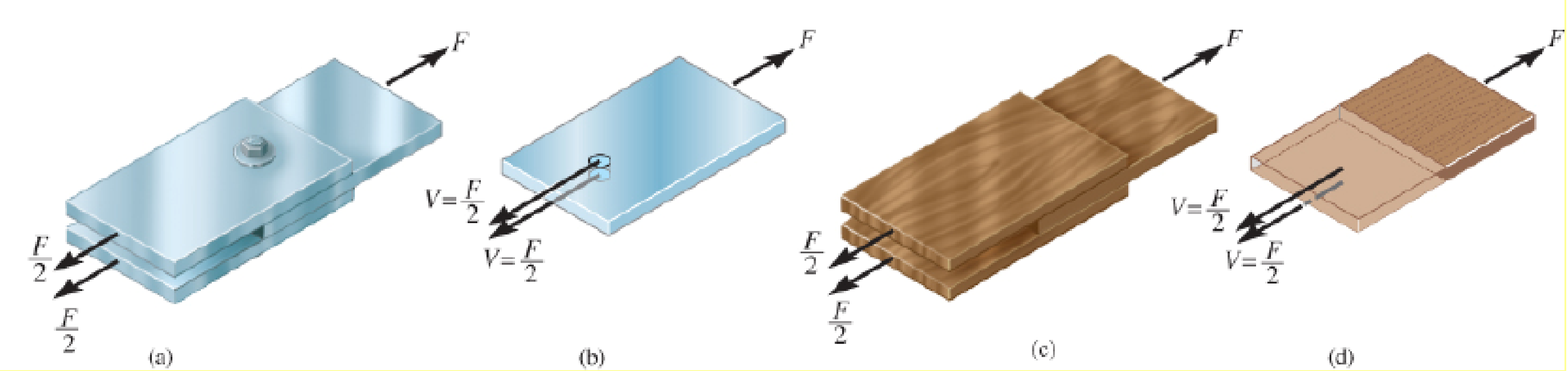
Direct shear stress

Double shear (Double Shear Connection)

- The joints shown below are examples of double-shear connections, **often called double lap joints.**
- For equilibrium, **x-sectional area of bolt** and **bonding surface between two members** subjected to double shear force, $V = F/2$
- Apply average shear stress equation to determine average shear stress acting on colored section in (d).



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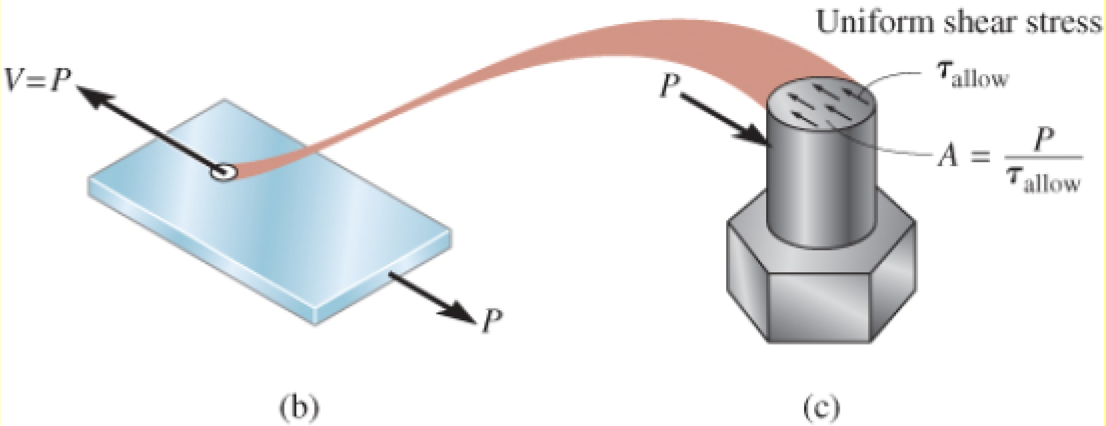
Direct shear stress

Cross-sectional area of a connecter subjected to shear



Assumption:

If bolt is loose or clamping force of bolt is unknown, assume frictional force between plates to be negligible.

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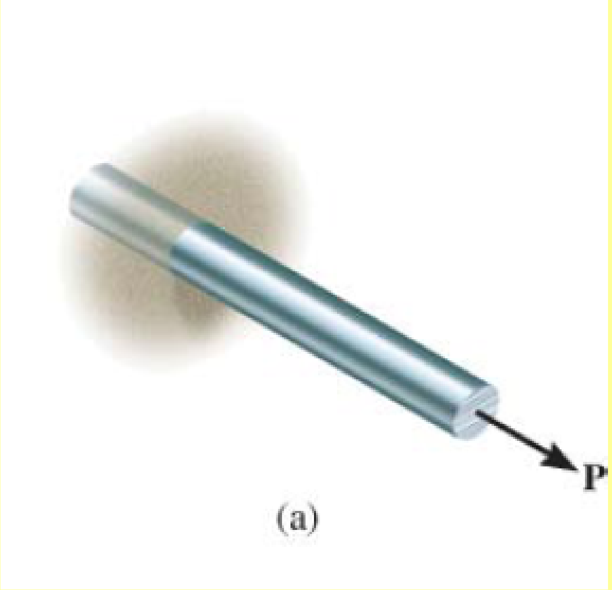
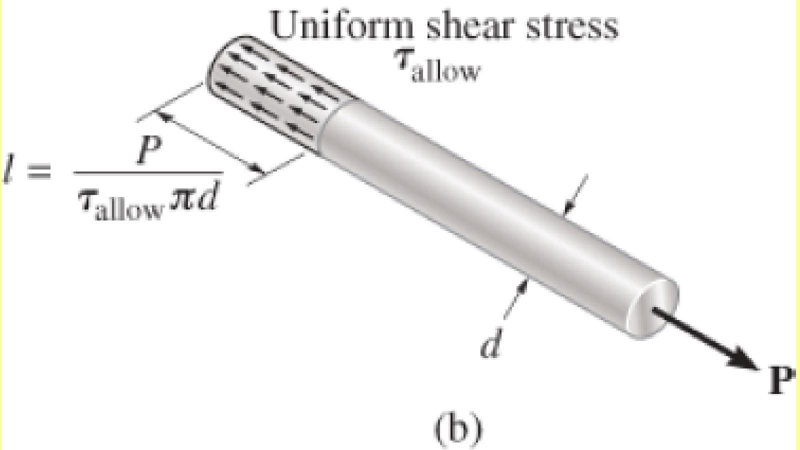
Direct shear stress

Required area to resist shear caused by axial load

- Although actual shear-stress distribution along rod difficult to determine, we assume it is *uniform*.
- Thus use $A = V / \tau_{allow}$ to calculate l, provided d and τ_{allow} is known.



(a)



(b)

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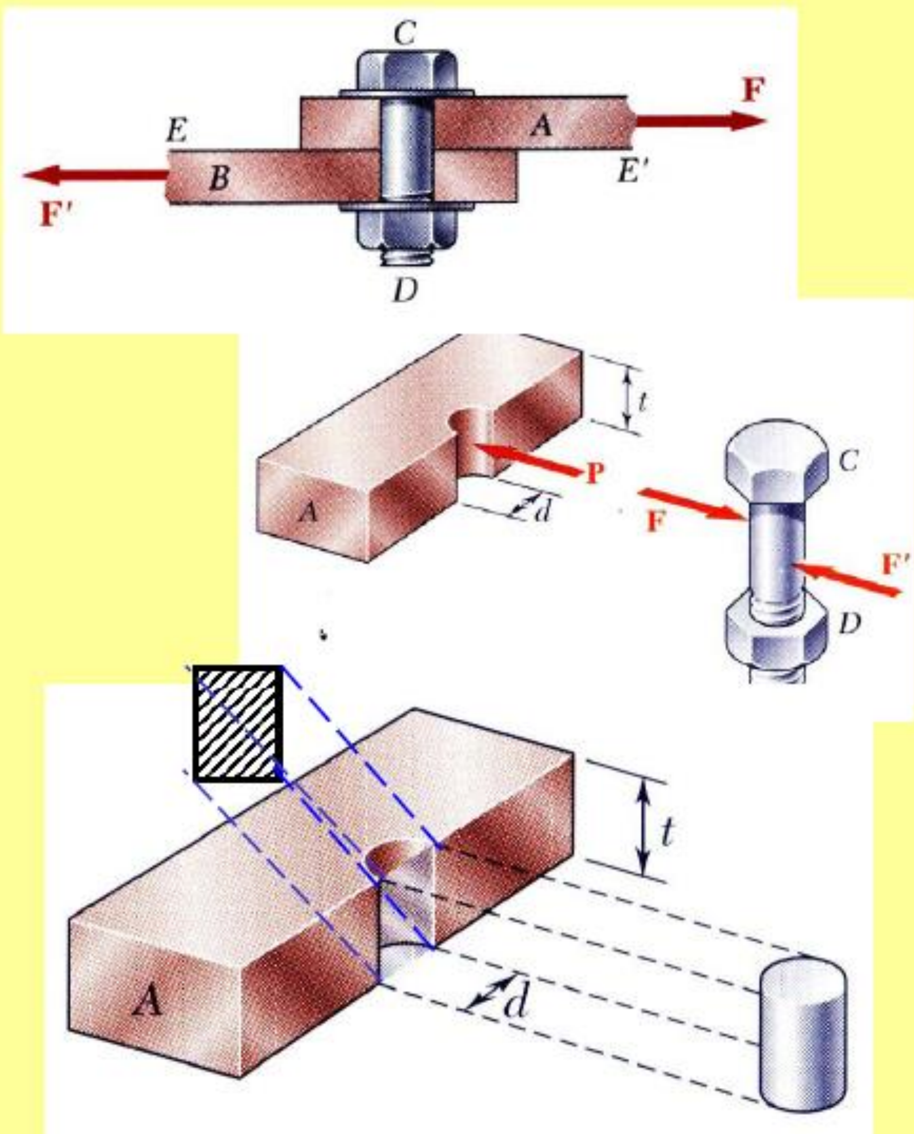
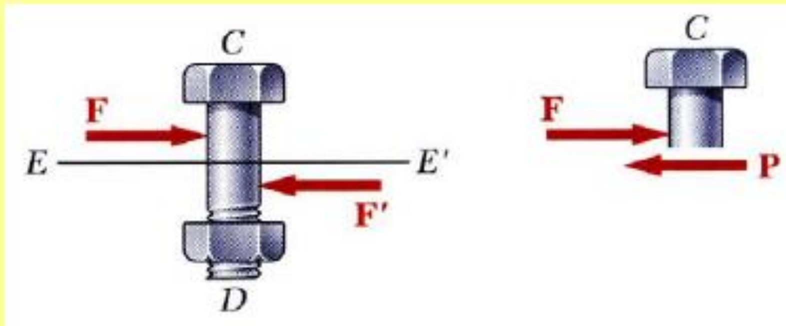
Bearing stress



- Bolts, rivets, and pins create stresses on the points of contact or ***bearing surfaces*** of the members they connect.

- **Corresponding average force intensity is called the bearing stress,**

$$\sigma_b = \frac{P}{A} = \frac{P}{t\,d}$$



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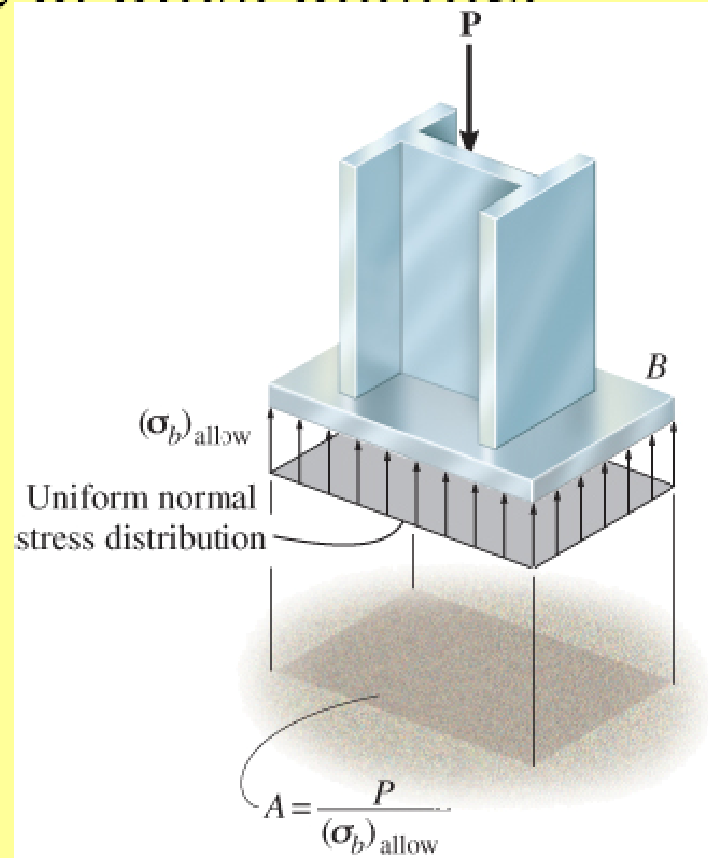
Bearing stress

Required area to resist bearing

- Bearing stress is normal stress produced by the *compression* of one surface against another

Assumptions:

1. $(\sigma_b)_{allow}$ of concrete < $(\sigma_b)_{allow}$ of base plate
2. Bearing stress is uniformly distributed between plate and concrete



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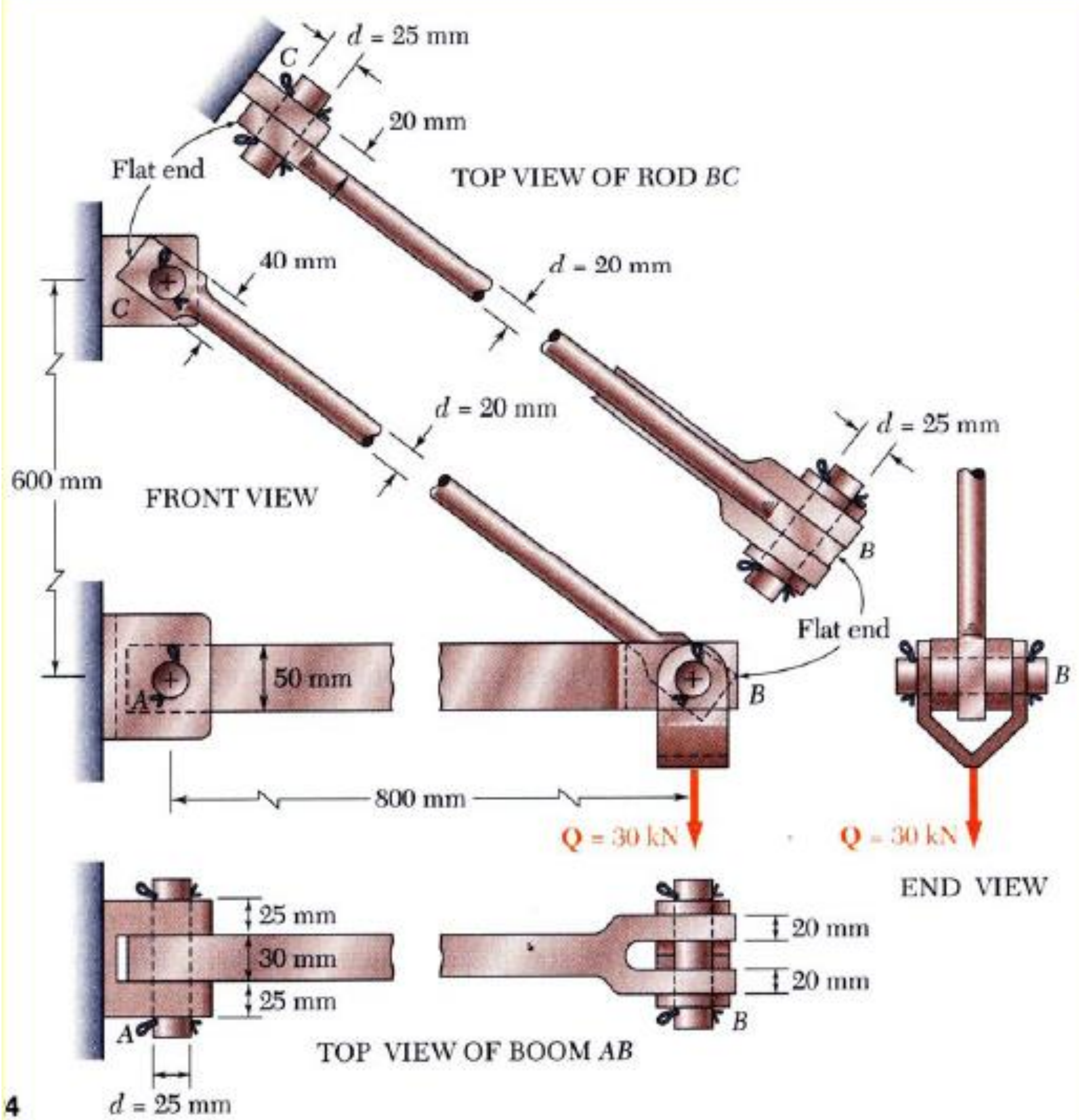
Stress analysis & design



- **Study** the **stresses** in the **members** and **connections** of the structure shown.

- **Calculate maximum normal stresses** in AB and BC, and the **shearing stress** and **bearing stress** at each pinned connection

- Knowing:
  $F_{AB} = 40$ kN (compression)
  $F_{BC} = 50$ kN (tension)

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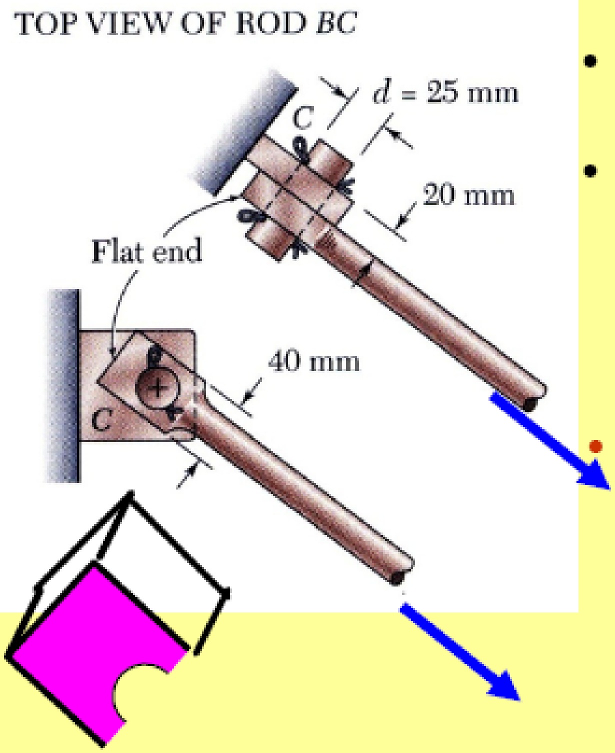
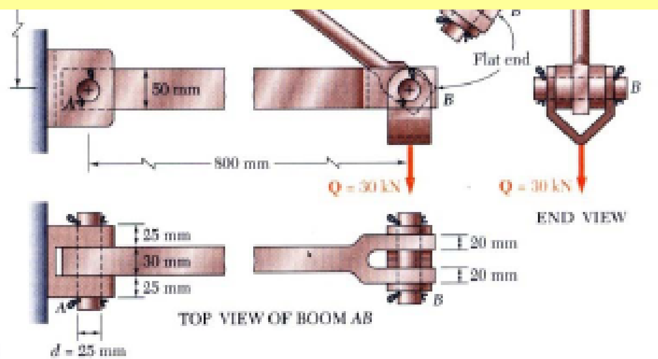
Stress analysis & design



- The rod is in tension with an axial force of 50 kN.
- At the rod center, the average normal stress in the circular cross-section ($A = 314 \times 10^{-6} m^2$) is $\sigma_{BC} = +159$ MPa. (**σ= F/ ($\pi d^2/4$) =50 000 x 4/ $\pi(0.02^2)$=159155000 kN/m^2**)
- **At the flattened rod ends, the smallest cross-sectional area occurs at the pin centerline,**

$$A = (20\,\text{mm})(40\,\text{mm} - 25\,\text{mm}) = 300 \times 10^{-6}\,\text{m}^2$$

$$\sigma_{BC,end} = \frac{P}{A} = \frac{50 \times 10^3\,N}{300 \times 10^{-6}\,\text{m}^2} = 167\,\text{MPa}$$



- The boom is in compression with an axial force of 40 kN and average normal stress of –26.7 MPa.
- **The minimum area sections at the boom ends are unstressed since the boom is in compression.**

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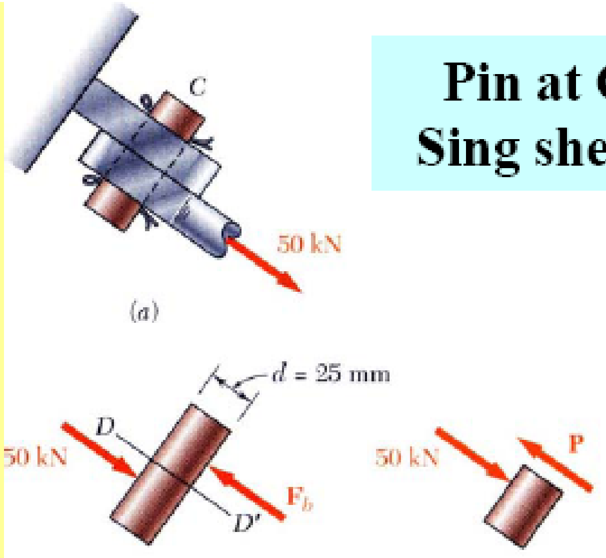
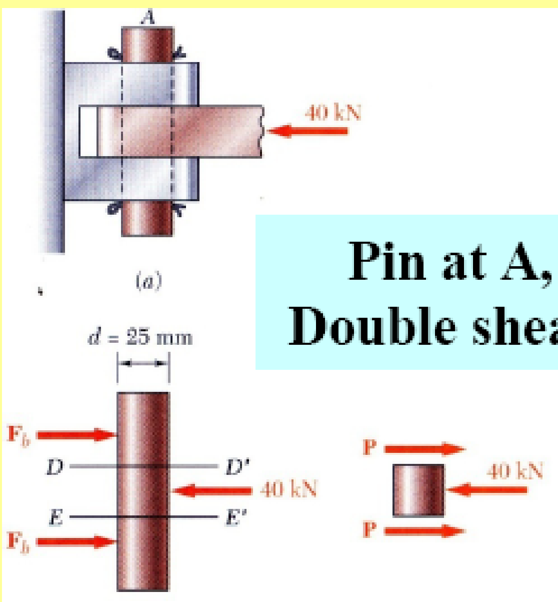
Stress analysis & design



**Pin at C,
Sing shear.**



**Pin at A,
Double shear.**

- The cross-sectional area for pins at *A*, *B*, and *C* ***are equal***,

$$A = \pi r^2 = \pi\left(\frac{25\,\text{mm}}{2}\right)^2 = 491\times10^{-6}\,\text{m}^2$$

- The force on **<u>the pin at *C*</u>** is equal to the force exerted by the rod *BC*,

$$\tau_{C,ave} = \frac{P}{A} = \frac{50\times10^3\,\text{N}}{491\times10^{-6}\,\text{m}^2} = 102\,\text{MPa}$$

- **<u>The pin at *A*</u>** is in **<u>double shear</u>** with a total force equal to the force exerted by the boom *AB*,

$$\tau_{A,ave} = \frac{P}{A} = \frac{20\,\text{kN}}{491\times10^{-6}\,\text{m}^2} = 40.7\,\text{MPa}$$

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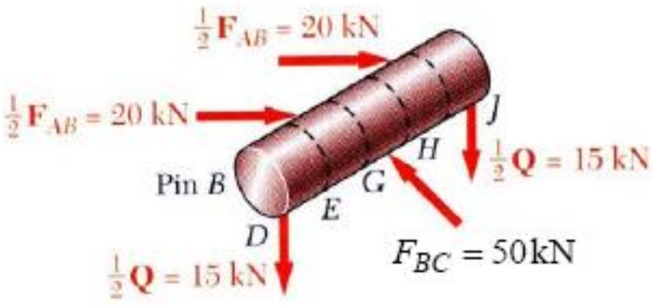
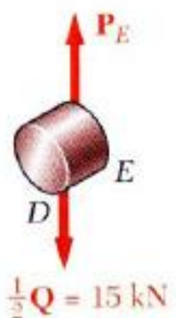
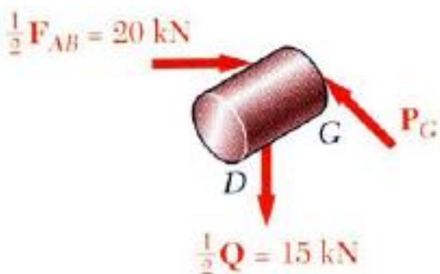
Stress analysis & design



(a)



(b)



- Divide the pin at B into sections to determine the section with the largest shear force,

$$P_E = 15\,\text{kN}$$

$$P_G = 25\,\text{kN} \ \ (\text{largest})$$

- Evaluate the corresponding average shearing stress,

$$\tau_{B,ave} = \frac{P_G}{A} = \frac{25\,\text{kN}}{491 \times 10^{-6}\,\text{m}^2} = 50.9\,\text{MPa}$$

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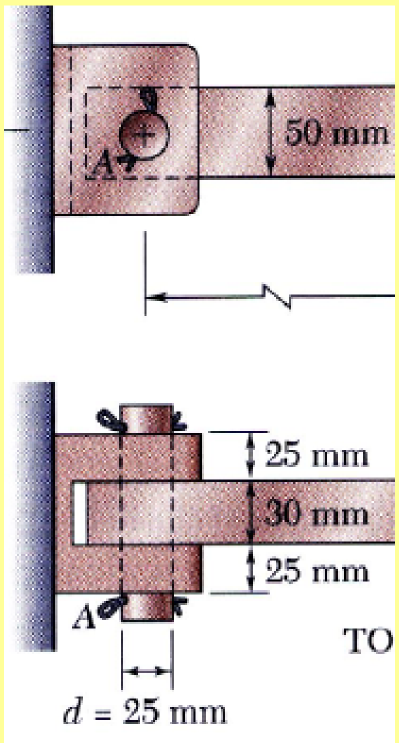
Stress analysis & design



- To determine the **bearing stress at A** in the boom *AB*, we have t = 30 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(30\,\mathrm{mm})(25\,\mathrm{mm})} = 53.3\,\mathrm{MPa}$$

- To determine the bearing stress at *A* in the bracket, we have t = 2(25 mm) = 50 mm and d = 25 mm,

$$\sigma_b = \frac{P}{td} = \frac{40\,\mathrm{kN}}{(50\,\mathrm{mm})(25\,\mathrm{mm})} = 32.0\,\mathrm{MPa}$$

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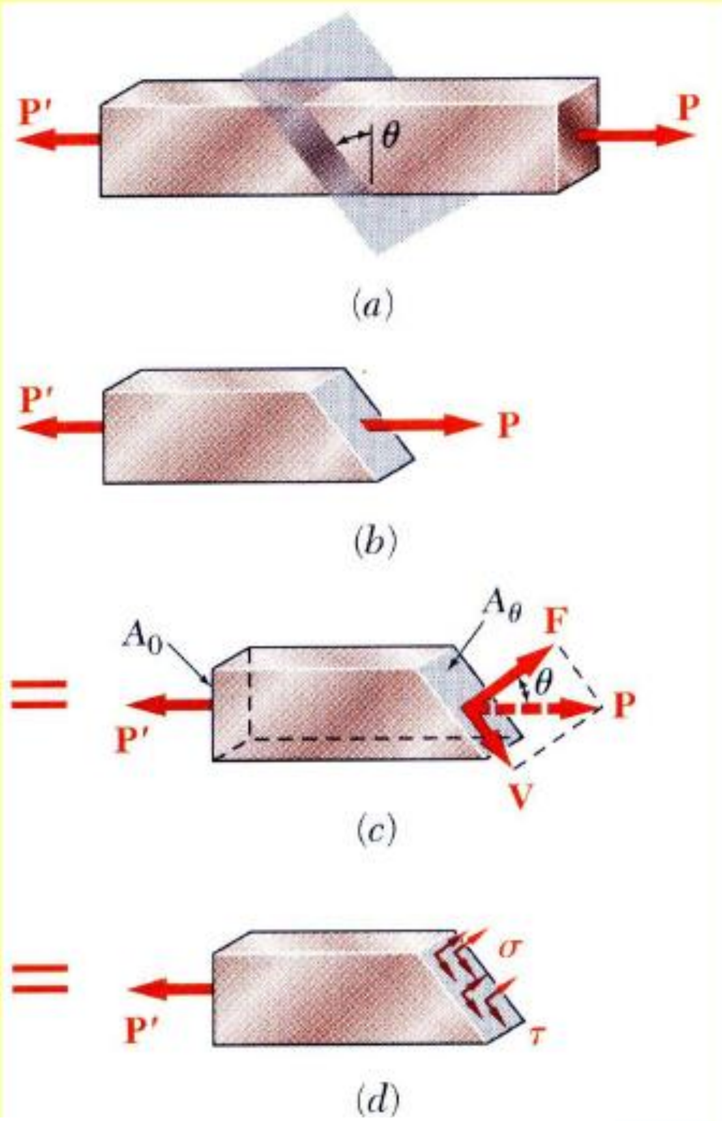
Stress on an oblique plane



- Pass a section through the member forming an angle θ with the normal plane.
- From equilibrium conditions, the distributed forces (stresses) on the plane must be equivalent to the force P.
- Resolve P into components normal and tangential to the oblique section,

$$F = P\cos\theta \qquad V = P\sin\theta$$

- The average normal and shear stresses on the oblique plane are

$$\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A_0/\cos\theta} = \frac{P}{A_0}\cos^2\theta$$

$$\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A_0/\cos\theta} = \frac{P}{A_0}\sin\theta\cos\theta$$

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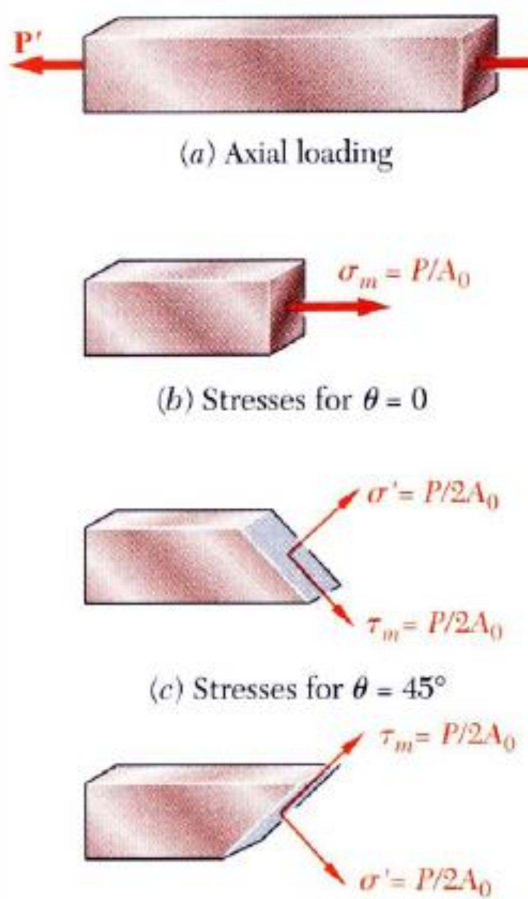
Stress on an oblique plane



(a) Axial loading

(b) Stresses for $\theta = 0$

(c) Stresses for $\theta = 45°$

(d) Stresses for $\theta = -45°$

- Normal and shearing stresses on an oblique plane

$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

- The maximum normal stress occurs when the reference plane is perpendicular to the member axis,

$$\sigma_m = \frac{P}{A_0} \qquad \tau' = 0$$

- The maximum shear stress occurs for a plane at $\pm$ 45° with respect to the axis,

$$\tau_m = \frac{P}{A_0}\sin 45 \cos 45 = \frac{P}{2A_0} = \sigma'$$

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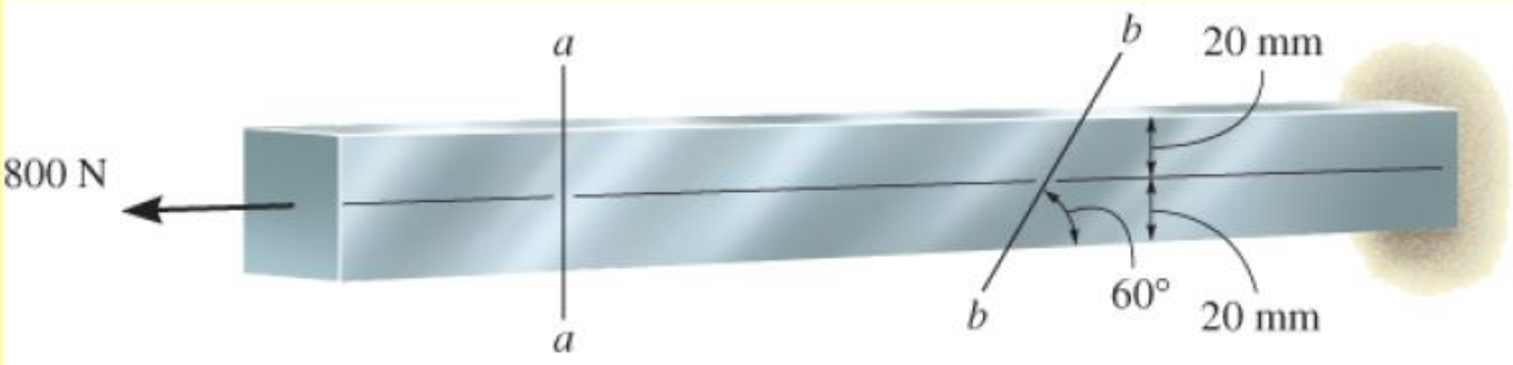
Stress on an oblique plane

Determine average **normal stress** and average **shear stress** acting along **(a)** section planes *a-a*, and **(b)** section plane *b-b*.



(a)

Depth and thickness = 40 mm

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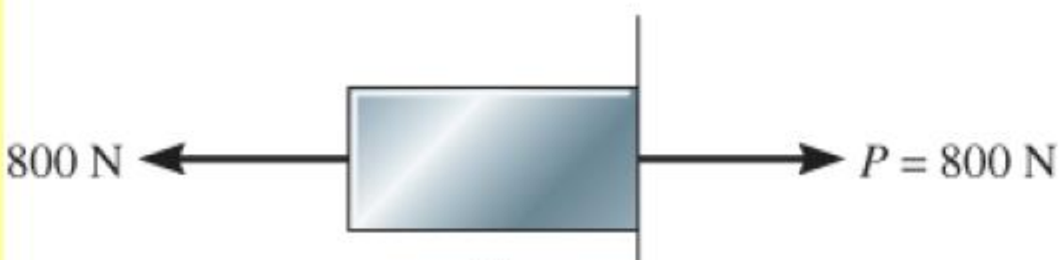
Stress on an oblique plane

Part (a): Internal loading

Based on free-body diagram, Resultant loading of axial force, $P = 800$ N



(b)

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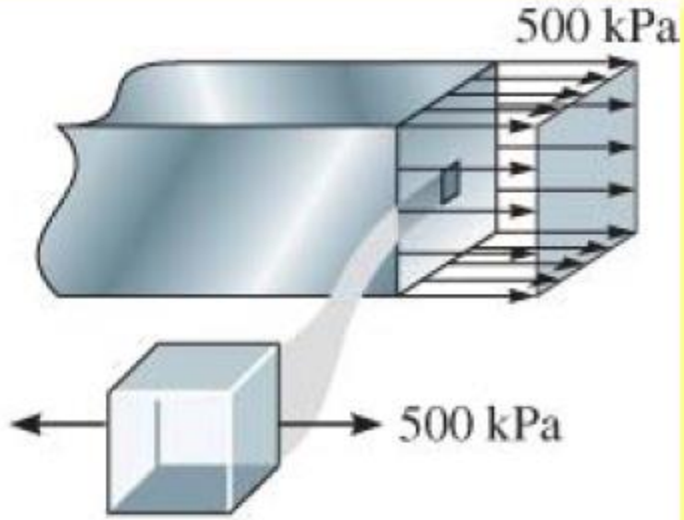
Stress on an oblique plane

Part (a): Average stress

Average normal stress, σ

$$\sigma = \frac{P}{A} = \frac{800\ \text{N}}{(0.04\ \text{m})(0.04\ \text{m})} = 500\ \text{kPa}$$



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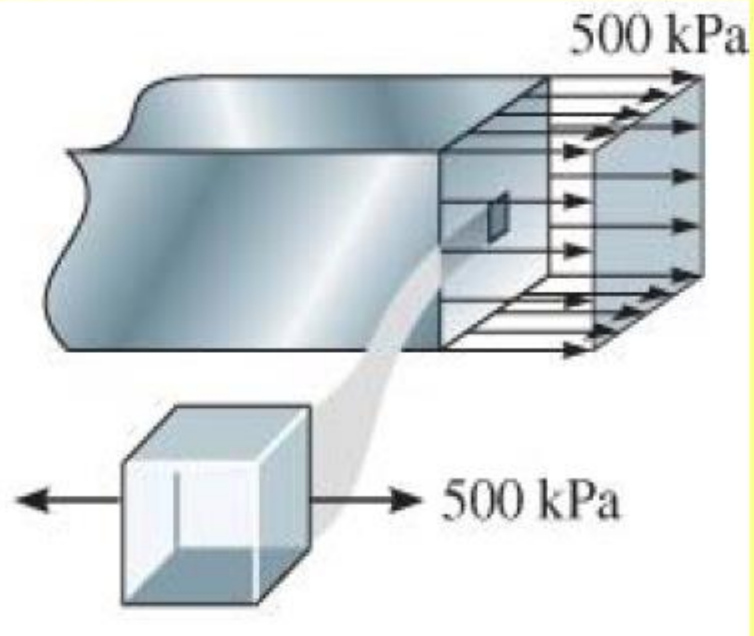
Stress on an oblique plane

Part (a): Internal loading

No shear stress on section, since shear force at section is zero.



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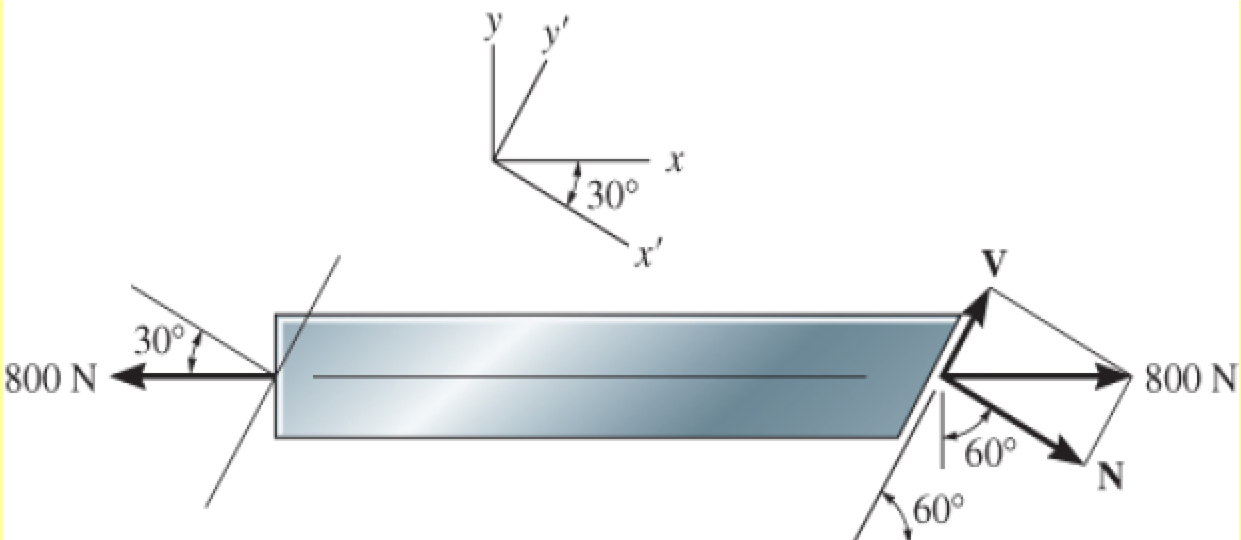
Stress on an oblique plane

Part (b): Internal loading

$$\xrightarrow{+} \sum F_x = 0; \quad -800\text{ N} + N\sin 60^\circ + V\cos 60^\circ = 0$$

$$+\uparrow \sum F_y = 0; \quad V\sin 60^\circ - N\cos 60^\circ = 0$$



(d)

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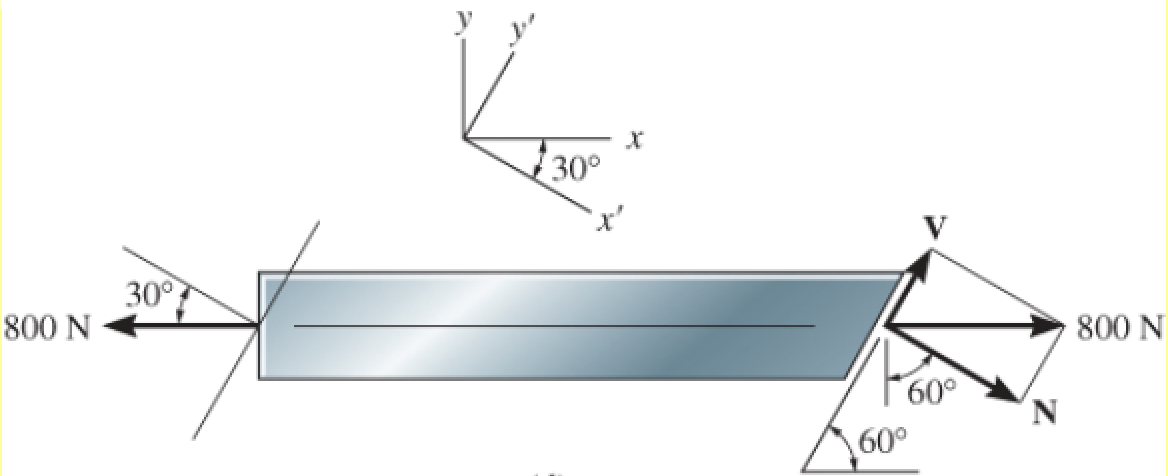
Stress on an oblique plane

Part (b): Internal loading

Or directly using x', y' axes,

$$\searrow^{+} \sum F_{x'} = 0; \qquad N - 800\text{ N}\cos 30^\circ = 0$$

$$\nearrow^{+} \sum F_{y'} = 0; \qquad V - 800\text{ N}\sin 30^\circ = 0$$



(d)

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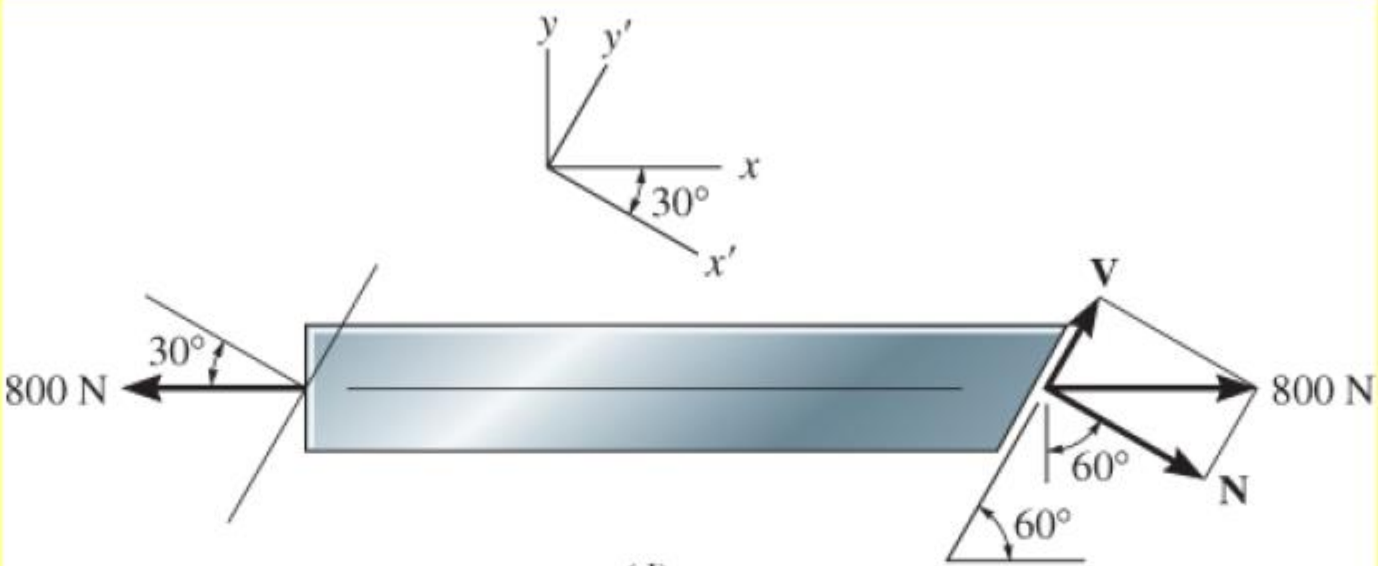
Stress on an oblique plane

Part (b) Average normal stress

$$\sigma = \frac{N}{A} = \frac{692.8\ \text{N}}{(0.04\ \text{m})(0.04\ \text{m}/\sin 60^\circ)} = 375\ \text{kPa}$$



(d)

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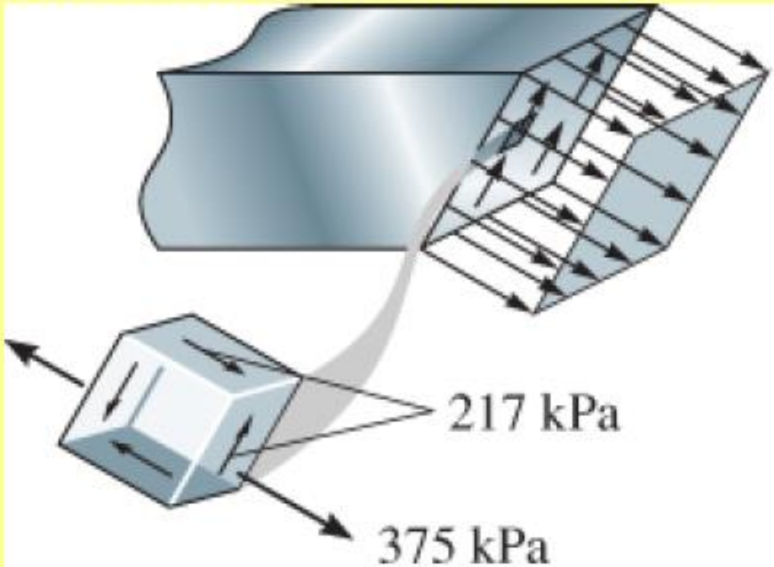
Stress on an oblique plane

Part (b): Average shear stress

$$\tau_{avg} = \frac{V}{A} = \frac{400\ \text{N}}{(0.04\ \text{m})(0.04\ \text{m}/\sin 60°)} = 217\ \text{kPa}$$

Stress distribution shown below



(e)

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Factor of safety & allowable stress

Structural members or machines must be designed such that the working stresses are less than the ultimate strength of the material.

$$FS = \text{Factor of safety}$$

$$FS = \frac{\sigma_u}{\sigma_{all}} = \frac{\text{ultimate stress}}{\text{allowable stress}}$$

Factor of safety considerations:

- uncertainty in material properties
- uncertainty of loadings
- uncertainty of analyses
- number of loading cycles
- types of failure
- maintenance requirements and deterioration effects
- importance of member to integrity of whole structure
- risk to life and property
- influence on machine function

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Factor of safety & allowable stress

- When designing a structural member or mechanical element, the stress in it must be restricted to safe level
- Choose an allowable load that is less than the load the member can fully support
- One method used is the factor of safety (F.S.)

$$\textbf{F.S.} = \frac{F_{\text{fail}}}{F_{\text{allow}}}$$

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Factor of safety & allowable stress

- If load applied is linearly related to stress developed within member, then F.S. can also be expressed as:

$$\textbf{F.S.} = \frac{\sigma_{\text{fail}}}{\sigma_{\text{allow}}} \qquad \textbf{F.S.} = \frac{\tau_{\text{fail}}}{\tau_{\text{allow}}}$$

- In all the equations, F.S. is chosen to be greater than 1, to avoid potential for failure
- Specific values will depend on types of material used and its intended purpose

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Factor of safety & allowable stress

- To determine area or dimensions of section subjected to a *normal force*, use

$$A = \frac{P}{\sigma_{allow}}$$

- To determine area or dimensions of section subjected to a *shear force*, use

$$A = \frac{V}{\tau_{\text{allow}}}$$

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